

Chapter one

Short Questions:

1. Prove the identity $\cosh^2 x + \sinh^2 x = \cosh 2x$ (2016)(Example 2 p.g 9)
2. Define explicit and Implicit function. (2019)
Explicit function “If y is easily expressed in term of the independent variable x , then y is called an explicit function of x. for example: $y = x^2 + 2x - 1$, $y = \sqrt{x-1}$ ” Symbolically it is written as $f(x)=y$

Implicit function “If x and y are so mixed up and y cannot be expressed in term of independent variable x, then y is called implicit function of x. For example $x^2 + xy + y^2 = 2$ and $\frac{xy^2 - y + 9}{xy}$ are implicit function of x and y. Symbolically it is written as $f(x,y)=0$

3. Determine whether the function $f(x) = x\sqrt{x^2 + 5}$ is even or odd. (2019)(E.x#1.1 Q9 iii)
4. Express the volume V of a cube as a function of area A of its base. (E.x#1.1 Q3 c)
5. Express the Area A of a circle as a function of its circumference.(2017) (E.x#1.1 Q3 b)
6. Find $\frac{f(a+h) - f(a)}{h}$ and simplify where $f(x)=\sin x$. (2015) (E.x#1.1 Q2 ii)
7. Determine the function is even or odd if $f(x) = x^{\frac{2}{3}} + 6$. (2015) (E.x#1.1 Q9 v)
8. Find the Domain and Range of the function $F(x) = \frac{x^2 - 16}{x - 4}$ $x \neq 4$. (E.x#1.1 Q4 viii)
9. Define Continuous function.

Continuous function “ A function f is said to be continuous at a number iff the following three conditions are satisfied:

$$\text{i) } f(c) \text{ is defined} \quad \text{ii) } \lim_{x \rightarrow c} f(x) \text{ exists} \quad \text{iii) } \lim_{x \rightarrow c} f(x) = f(c) \text{ ”}$$

10. Prove that $\sinh 2x = 2 \sin h x \cosh x$. (E.x#1.1 Q8 i)
11. Find Domain and Range of the given function $f(x) = |x - 3|$.(E.x#1.1 Q4 iv)
12. Define Even and odd function with example.

Even Function: “A function f is said to be even if $f(-x) = f(x)$ for every number x in the domain of f.

For example: $f(x) = x^2$ here $f(-x) = (-x)^2 = x^2 = f(x)$ ”

Odd Function: “A function f is said to be odd if $f(-x) = -f(x)$ for every number x in the domain of f.

For example: $f(x) = x^3$ here $f(-x) = (-x)^3 = -x^3 = -f(x)$ ”

13. If $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2}$ find $g \circ f(x)$. (2017)(E.x 1.2 Q1 iii)
14. If $f(x) = (-x + 9)^3$ then verify that $f^{-1}(f(x)) = x$. (2015) (2017) (E.x 1.2 Q2 iii)
15. Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$ (2017)(Example 7 p.g 26)
16. Prove that $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$ (2019)(pg#20 1.5.2)
17. Prove that: $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \text{Log}_e a$. (pg#23 1.5.6)
18. Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$. (2012) (E.x 1.3 Q2 viii)
19. Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$ (2012) (E.x 1.3 Q3 xii)
20. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$ (2017)(E.x 1.3 Q3 viii)
21. Express each limit in term of e, $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n$. (2014) (E.x 1.3 Q4 iii)
22. Express each limit in term of e, $\lim_{n \rightarrow \infty} \left(1 + \frac{4}{n}\right)^n$ (2014) (E.x 1.3 Q4 v)
23. Evaluate $\lim_{n \rightarrow \infty} \left(\frac{x}{1+x}\right)^x$ in term of e. (E.x 1.3 Q4 iv)
24. Evaluate $\lim_{x \rightarrow 0} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}}$ where $x > 0$. (E.x 1.3 Q4 xi)
25. Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\theta}$ (E.x 1.3 Q3 ix)
26. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$ (E.x 1.3 Q3 ii)
27. Evaluate $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$ (E.x 1.3 Q3 iv)

Long Question:

28. Prove that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$. (2014,2017)(pg 23 1.5.5)

29. Let $f(x) = \frac{2x+1}{x-1}$; $x \neq 1$, find $f^{-1}(x)$ and verify that $f \circ f^{-1}(x) = x$ (2016)(E.x #1.2 Q.2 iv)

30. Evaluate the following $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$. (2016)(E.x 1.3 Q3 iv)

31. Evaluate $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$. (2015,2017) (E.x 1.3 Q3 v)

32. Discuss the continuity of function $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$ at $x=3$ (2016)(Example #4 p.g 30)

33. If $F(x) = \begin{cases} x-1 & \text{if } x < 3 \\ 2x+1 & \text{if } x \geq 3 \end{cases}$ discuss continuity at $x=3$ (2016)(Example 5 p.g30)

34. If $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$ Discuss the continuity of $f(x)$ at $x = 1$. (E.x 1.4 Q2 ii)

35. If $F(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$ discuss continuity at $x=2$ and $x=-2$. (2013)(E.x 1.4 Q3)

36. If $F(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x+9 & \text{if } x > 3 \end{cases}$ discuss continuity at $x=3$ (2015)(E.x 1.4 Q4)

37. If $F(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$

find the value of K if function is continuous.(2014) (E.x 1.4 Q6)