Mathematics Question Bank (Inter Part – II)

Chapter Wise Long Questions

Chapter No. 1 (Functions and Limits)

1. Evaluate $\lim_{x\to 0} \frac{1-cosx}{sin^2 x}$ 17 Grp I,

(Pg#27)

2. Evaluate $\lim_{x\to 0} \frac{sinx}{x} = 1$

3. Evaluate $\lim_{\theta\to 0} (\frac{1-cosp\theta}{1-cosq\theta})$ 16 Grp I, 12 Grp I,

(Pg#27)

4. Evaluate $\lim_{\theta\to 0} \frac{tan\theta-sin\theta}{sin^3\theta}$ 13 Grp II,

(Pg#27)

5. Find the values of m and n, so that given function f is continuous at $x = 3$.19 Grp I, 15 Grp I,

If $f(x) = \begin{cases} mx & if\ x < 3 \\ n & if\ x = 3 \\ -2x + 9 & if\ x > 3 \end{cases}$ (Pg#32)

6. Discuss the continuity of $f(x)$ at $x = 2$ and $x = -2$. 13 Grp I,

If $f(x) = \begin{cases} 3x & if\ x \leq -2 \\ x^2 & if -2 < x < 2 \\ 3 & if\ x \geq 2 \end{cases}$ (Pg#31)

7. If $f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x \neq 2 \\ k & x = 2 \end{cases}$ (Pg#32)

Find the value of k so that f is continuous at $x = 2$. 18 Grp I, 15 Grp II, 14 Grp I,

8. Let $f(x) = \frac{2x+1}{x-1}; x \neq 1$, find $f^{-1}(x)$ and verify $fof^{-1}(x) = x$ 16 Grp II,

9. Prove $\lim_{x\to\infty} \left(1 + \frac{1}{n}\right)^n = e$ 14 Grp II, (Pg#22)

10. Prove that $\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log_e a$ 18 II, (Pg#23)

11. Prove that $\lim_{x\to 0}\frac{sinx}{x} = 1$ 12 II,

Chapter No. 2 (Differentiation)

1. Differentiate $\frac{x^2+1}{x^2-1}$ w.r.t. $\frac{x-1}{x+1}$ 14 Grp I,
2. Differentiate $x^2+\frac{1}{x^2}$ w.r.t. $x-\frac{1}{x}$ 15 Grp II,
3. Differentiate $cos\sqrt{x}$ from the first principle. 15 Grp I,
4. Differentiate $sin\sqrt{\frac{1+2x}{1+x}}$ w.r.t x 16 Grp I,
5. Find $\frac{dy}{dx}$ if $x=a(cost+sint), y=a(sint-tcost)$ 17 Grp I,
6. Find two positive integers whose sum is 9 and the product of one with the square of the other will be maximum. 11 Grp I, 10 Grp II,

   (Pg#114)
7. If $x=sin\theta$, $y=sinm\theta$, Show that $(1-x^2)y_2-xy_1+m^2y=0$ 13 Grp II,
8. If $y=(cos^{-1}x)^2$, prove that $(1-x^2)y_2-xy_1-2=0$ 12 Grp II,
9. If $y=e^x.sinx$, then prove that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$ 19 Grp I,

   (Pg#95)
10. Prove that $y\frac{dy}{dx}+x=0$ if $x=\frac{1-t^2}{1+t^2}, y=\frac{2t}{1+t^2}$. 18 Grp II, 12 Grp I, (Pg#71)
11. Show that $\cos(x+h)=cosx-h\sin x-\frac{h^2}{\underline{|2}}\ cosx+\frac{h^3}{\underline{|3}}sinx+\ldots$. And evaluate $cos61^o$ 10 Grp I,
12. Show that $\frac{dy}{dx}=\frac{y}{x}$ if $\frac{y}{x}=Tan^{-1}\frac{y}{x}$ 16 Grp II,
13. Show that $y=\frac{lnx}{x}$ has maximum value at $x=e$. 14 Grp II, 11 Grp II, (Pg#113)
14. Show that $y=x^x$ has a maximum value at $=\frac{1}{e}$ 18 Grp I, (Pg#113)

Chapter No. 3 (Integration)

1. Evaluate $\int \left(\frac{1-sinx}{1-cosx}\right) e^x \, dx$ 16 Grp II, 11 Grp II, (Pg#145)

2. Evaluate $\int \left(\frac{1-sinx}{1-cosx}\right) e^x dx$ 16 Grp II, 11 Grp II, (Pg#145)

3. Evaluate $\int \frac{\sqrt{2}}{sinx+cosx} \, dx$ 19 Grp I, (Pg#137)

4. Evaluate $\int \frac{e^x(1+sinx)}{(1+cosx)} \, dx$ 12 Grp II, 11 Grp I, (Pg#143)

5. Evaluate $\int \frac{1}{x(x^3-1)} \, dx$ 15 Grp II.

6. Evaluate $\int cos^3 x\sqrt{sinx}\, dx$, $(sinx > 0)$ 10 Grp I, (Pg#133)

7. Evaluate $\int cosec^3 x \, dx$ 13 Grp II, (Pg#144)

8. Evaluate $\int \frac{cosx}{sinx \ln sinx} \, dx$ 16 Grp I, (Pg#163)

9. Evaluate $\int \frac{dx}{\frac{1}{2}sinx+\frac{\sqrt{3}}{2}cosx}$ 12 Grp I,

10. Evaluate $\int e^{2x} cos3x \, dx$ 18 Grp I, (Pg#144)

11. Evaluate $\int tan^3 x \sec x \, dx$ 17 Grp I, (Pg#144)

12. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosx}{sinx(2+sinx)} dx$ 10 Grp I,

13. Evaluate $\int_0^{\frac{\pi}{4}} cos^4 t \, dt$ 18 Grp II, 12 Grp I, (Pg#163)

14. Evaluate $\int_0^{\frac{\pi}{6}} cos^3\theta d\theta$ 10 Grp II, (Pg#163)

15. Evaluate $\int_0^{\pi/4} \frac{sinx-1}{cos^2 x} dx$ 15 Grp I,

16. Evaluate $\int_0^{\pi/4} \frac{sec\theta}{sec\theta+cos\theta} d\theta$ 16 Grp II, (Pg#163)

17. Evaluate $\int_{-1}^{2} (x + |x|) \, dx$ 14 Grp II,

18. Evaluate $\int_2^3 \frac{3x^2-2x+1}{(x-1)(x^2+1)} dx$ 16 Grp I,

19. Evaluate $\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$ 13 Grp II, (Pg#163)

20. Evaluate the indefinite integral $\int \sqrt{a^2 - x^2}\ \ dx$ 14 Grp I, (Pg#144)

21. Find the area between the x-axis and the curve $y = \sqrt{2ax - x^2}; a > 0$. 17 Grp I, 15 Grp II,

22. Find the area bounded by the curve $y = x^3 - 4x$ and x-axis. 18 Grp I, 12 Grp II,

23. Show that $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + c$ 18 Grp II, 15 Grp I, 10 Grp I,

24. Solve the differential equation $(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$ 19 Grp I, 11 Grp I,

25. Solve the following differential equation $(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$ 11 Grp I, (Pg#177)

26. Solve the following differential equation $1 + cos\ x\ tan\ y\ \frac{dy}{dx} = 0$ 14 Grp I, (Pg#178)

27. Solve the following differential equation $x\ dy + y(x - 1)dx = 0$ 11 Grp II,

28. Use differentials to approximate the values of $(31)^{1/5}$ 14 Grp II, (Pg#123)

$y = \sqrt{2ax - x^2}$ when $a > 0$.

Chapter No. 4 (Intro. to Analytic Geometry)

1. Find a joint equation of the straight lines through the origin perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$. 15 Grp II,

2. Find an equation of the perpendicular bisector joining the points A $(3\,,5)$ and B $(9\,,8)$ 12 Grp I,

3. Find an equation of the perpendicular bisector of the segment joining the points A $(3\,,5)$ and B $(9\,,8)$. 19 Grp I, 14 Grp I, 12 Grp I,

4. Find equations of the sides, altitudes and medians of the triangle whose vertices are A $(-3\,,2)$, B $(5\,,4)$ and C $(3\,,-8)$. 11 Grp I, (Pg#216)

5. Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x-intercept and y-intercept of each is 3. 15 Grp I, 12 Grp II, (Pg#218)

6. Find h such that the points A $\left(\sqrt{3}\,,-1\right)$, B $(0\,,2)$, C $(\,h\,,-2)$ are the vertices of a right triangle with right angle at the vertex A. 14 Grp II,

7. Find interior angles of a triangle whose vertices are A $(6\,,1)$, B $(2\,,7)$ and C $(-6\,,7)$. 16 Grp II,

8. Find the condition that the line $y = mx + c$ touches the circle $x^2 + y^2 = a^2$ at a single point. 18 Grp I,

9. Find the condition that the lines $y = m_1x + c_1;\ m_2x + c_2; y = m_3x + c_3$ are concurrent. 13 Grp II, (Pg#223)

10. Find the distance between the given parallel lines. Also find equation of parallel lying midway between them. $3x - 4y + 3 = 0$ and $3x - 4y + 7 = 0$ 16 Grp I, (Pg#217)

11. Find the equations of altitudes of ΔABC whose vertices are A $(-3\,,2)$, B $(5\,,4)$ and C $(3\,,-8)$ 11 Grp I,

12. Find the interior angles of a triangle whose vertices are A $(6\,,1)$, B $(2\,,7)$, C $(-6\,,-7)$. 16 Grp II,

13. Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$. 16 Grp I,

14. Find the lines represented by each of the following and also find measure of the angle between them $x^2 + 2xy\ sec\alpha + y^2 = 0$ 19 Grp I, (Pg#228)

15. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long. 18 Grp II, 13 Grp II, 12 Grp II, 11 Grp I,

16. Prove that the line segments joining the mid-points of sides of quadrilateral taken in order form a parallelogram. 15 Grp II,

17. Prove that the midpoint of the hypotenuse of a right triangle is the circumcenter of the triangle. 11 Grp II,

18. The points A $(-1\,,2)$, B $(6\,,3)$ and C $(2\,,-4)$ are vertices of a triangle. Show the line joining the midpoint D of AB and the midpoing E of AC is parallel to BC and $DE = \frac{1}{2}$. 18 Grp II,

19. The three points A $(7\,,-1)$, B $(-2\,,2)$ and C $(1\,,4)$ are consecutive vertices of a parallelogram, find the fourth vertex. 18 Grp I,

20. The vertices of a triangle are A $(-2\,,3)$, B $(-4\,,1)$ and C $(3\,,5)$. Find the circumcircle of the triangle. 17 Grp I,

Chapter No. 5 (Linear Inequalities and Linear Programming)

1. Graph the feasible region of system of linear inequalities and find the corner points. 18 Grp II,

(Pg#243)

$2x + 3y \leq 18$, $x + 4y \leq 12$, $3x + y \leq 12$ $x \geq 0, y \geq 0$

2. Graph the feasible region of system of linear inequalities and find the corner points. 12 Grp I,

$3x + 7y \leq 21$, $2x - y \leq -3$, $y \geq 0$

3. Shade the feasible region and also find the corner points of: 15 Grp II,

$2x - 3y \leq 6$, $2x + 3y \leq 12$, $x \geq 0\ , y \geq 0$

4. Minimize $z = 2x + y$ subject to the constraints. 18 I, 14 Grp I,

(Pg#248)

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$

5. Graph the feasible region of system of linear inequalities and find the corner points.

19 Grp I, (Pg#243)

$x + y \leq 5$; $-2x + y \leq 2$; $y \geq 0$

6. Graph the feasible region of system of linear inequalities and find the corner points.

14 Grp II, 13 Grp II,

$2x - 3y \leq 6$; $2x + y \geq 2$; $y \geq 0\ , y \geq 0$

7. Minimize f $(x\ , y) = x + 3y$ subject to constraint. 17 Grp I, 16 Grp I,

$2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0, y \geq 0$

8. Minimize f $(x\ , y) = 2x + 3y$ subject to constraint. 16 Grp II, 11 Grp II,

$2x + y \leq 8$; $x + 2y \leq 14$; $x \geq 0, y \geq 0$

9. Find the minimum value of $\phi\,(x\,,y) = 4x + 6y$ under the constrains: 12 Grp II,

$2x - 3y \leq 6$, $2x + y \geq 2$, $2x + 3y \leq 12$ $x \geq 0, y \geq 0$

10. Minimize the function $z = 3x + y$ subject to the constrains: 11 Grp I,

$3x + 5y \geq 6$, $x + 6y \geq 9$, $x \geq 0, y \geq 0$

Chapter No. 6 (Conic Sections)

1. Find an equation of parabola having its focus at the origin and directrix parallel to y-axis. 18 Grp II, (Pg#274)
2. Find the centre, foci, eccentricity, vertices and equation of directices of $\frac{y^2}{4} - x^2 = 1$.
   18 Grp II, (Pg#298)
3. Find x so that points A $(1, -1, 0)$. B $(-2, 2, 1)$ and C $(0, 2, x)$ from triangle with right angle at C. 18 Grp I,
4. Find the coordinates of the points of intersection of the line $2x + y + 5 = 0$ and the circle $x^2 + y^2 + 2x - 9 = 0$. Also find the length of intercepted chord. 19 Grp I,
5. Find equation of parabola with elements directrix : $x = -2$, focus $(2, 2)$. 19 Grp I,
6. Find an equation of parabola whose focus is F $(-3, 4)$, directrix line is $3x - 44y + 5 = 0$. 15 Grp I,
7. Find the focus, vertex and the directrix of the parabola $x^2 - 4x - 8y + 4 = 0$. 15 Grp II,
8. Write an equation of the parabola with axis $y = 0$ and passing through $(2, 1)$ and $(11, -2)$. 16 Grp II,
9. Show that the line $3x - 2y = 0$ and $2x + 3y - 13 = 0$ are tangents to the circle $x^2 + y^2 + 6x - 4y = 0$. 17 Grp I, 16 Grp II,
10. Show that the equation $9x^2 - 18x + 4y^2 + 8y - 23 = 0$ represent an ellipse. Find its elements (foci, vertices and directrices) 16 Grp I,
11. Show that the equation $x^2 + 16x + 4y^2 - 16y + 76 = 0$ represent an ellipse. Find its foci eccentricity, vertices and directrices. 14 Grp I,
12. Write equations of tangent lines to the circle $x^2 + y^2 + 4x + 2y = 0$ down from the point P $(-1, 2)$. Also find the tangential distance. 14 Grp I,
13. Prove that in any triangle ABC by vector method $a^2 = b^2 + c^2 - 2bc \cos A$. 16 Grp I,

14. Find equation of ellipse having vertices $(0\,,\pm5)$ and eccentricity $\frac{3}{5}$. 14 Grp II,

15. Find an equation of the circle passing through the point $(-2\,,-5)$ and touching the line $3x+4y-24=0$ at the point $(4\,,3)$. 14 Grp II,

16. Find the foci, vertex and directrix of the parabola $y=6x^2-1$. 13 Grp II,

17. Find equations of the tangents to the circle $x^2+y^2=2$ 13 Grp II,

18. Find an equation of an ellipse with Foci $(-3\sqrt{3}\,,0)$ and vertices $(\pm6\,,0)$ 12 Grp II,

19. Find equation of the circle passing through A $(a\,,0)$, B $(0\,,b)$ and C $(0\,,0)$ 12 Grp II,

20. Find an equation of the parabola with focus $(1\,,2)$ and vertex $(3\,,2)$. 12 Grp I,

21. Write an equation of the circle that passes through the point A $(a\,,0)$, B $(0\,,b)$, C $(0\,,0)$.12 Grp I,

22. Write an equation of the circle that passes through the points A $(4\,,5)$, B $(-4\,,-3)$, and C $(8\,,-3)$. 11 Grp I,

23. Find the equation of parabola with focus at point $(a\cos\alpha\,,a\sin\alpha)$ and directrix as $x\,cos\alpha+y\,sin\alpha+a=0$. 11 Grp II,

24. Find equation of the circle of radius 2 and tangent to the line $x-y-4=0$ at A $(1\,,-3)$ 15 Grp II,

Chapter No. 7 (Vectors)

1. Find the value of α, in the coplanar vectors $\alpha\underline{i} + \underline{j}, \underline{i} + \underline{j} + 3\underline{k},$ and $2\underline{i} + \underline{j} - 2\underline{k}$. 18 II, (Pg#362)

2. If $\underline{a} = 3\underline{i} - \underline{j} - 4\underline{k}$; $\underline{b} = -2\underline{i} - 4\underline{j} - 3\underline{k}$ and $\underline{c} = \underline{i} + 2\underline{j} - \underline{k}$, then find a unit vector parallel to $-3\underline{a} - 2\underline{b} + 4\underline{c}$. 16 Grp II, 12 Grp I,

3. (Example) Find the volume of the tetrahedron whose vertices are A $(2, 1, 8)$, B $(3, 2, 9)$, C $(2, 1, 4)$ and D $(3, 3, 10)$. 18 Grp I, 16 Grp I, Grp II, (Pg#362)

4. Prove that $\sin(\alpha - \beta) = sina.cos\beta - cos\alpha sin\beta$ by method of vectors. 19 Grp I,

5. Find the volume of the tetrahedron with the vertices of A $(0, 1, 2)$, B $(3, 2, 1)$, C $(1, 2, 1)$ and D $(5, 5, 6)$. 15 Grp I, 14 Grp II, 13 Grp II,

6. Find the constant a such that the vectors are coplanar $\underline{i} - \underline{j} + \underline{k}, \underline{i} - 2\underline{j} - 3\underline{k},$ and $3\underline{i} - a\underline{j} + 5\underline{k}$. 15 Grp II,

7. The position vectors of the points A, B, C and D are $2\underline{i} - \underline{j} + \underline{k},\ 3\underline{i} + \underline{j},\ 2\underline{i} + 4\underline{j} - 2\underline{k}$ and $-\underline{i} + 2\underline{j} + \underline{k}$ respecetively. Show that AB is parallel to CD. 14 Grp II,

8. A force of magnitude 6 units acting parallel to $2\underline{i} - 2\underline{j} + \underline{k}$ displaces the point of application from $(1, 2, 3)$ to $(5, 3, 7)$. Find the work done. 14 Grp I,

9. Prove by using vectors that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long. 14 Grp I,

10. If $\underline{a} + \underline{b} + \underline{c} = 0$ then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ 12 Grp II,

11. A force $\underline{F} = 4\underline{i} - 3\underline{k}$ passes through the point A $(2, -2, 5)$. Find the moment of the force about the point B $(1, -3, 1)$. 12 Grp I,

12. Find a unit vector perpendicular to both vectors $\underline{a}$ and $\underline{b}$ where $\underline{a} = -\underline{i} - j - k$ and $\underline{b} = 2\underline{i} - 3\underline{j} + 4\underline{k}$. 11 Grp II,

13. If $\underline{a} = 3\underline{i} - j - 4\underline{k}, \underline{b} = -2\underline{i} - 4\underline{j} - 3\underline{k}$ and $\underline{c} = \underline{i} + 2\underline{j} - \underline{k}$ find a unit vector parallel to $3\underline{a} - 2\underline{b} + 4\underline{c}$. 11 Grp II,