LONG QUESTIONS

Question Bank

Chapter 2:

Q.1 Prove that p $\vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Q.2 Convert (A$\cup B) \cup C = A \cup (B \cup C)$ into logical form and prove it by constructing the truth table.

Q.3 Give the logical proof of De Morgan's Law.

Q.4 Convert the theorem (A$\cup B)' = A' \cup B'$ to logical statement and prove them by constructing truth tables.

Q.5 Show that the set $\{ 1, \omega, \omega^2\}, \omega^3 = 1$, is an Abelian group w.r.t ordinary multiplication.

Q.6 Prove that 2x2 non singular matrices over the real field form a non-abelian group under multiplication.

Q.7 Consider the set S={1,-1,i-i}.Set up its multiplication table and show that the set S

is an abelian group under multiplication.

Q.8 Give logical proofs of the following theorems

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $(A \cup B)' = A' \cap B'$

Q.9 If a,b are elements of a group G, solve the following equations:

i) $ax = b$ $\quad ii)\ xa = b$

Chapter 3:

Q.1 Find x and y if $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$

Q.2 Solve the following system of linear equations: $3x - 5y = 1$; $-2x + y = -3$

Q.3 Solve the following matrix equation for A: $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} A - \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$

Q.4 Show that $\begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix} = 0$

Q.5 Show that $\begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix} = \lambda^2(a+b+c+\lambda)$

Q.6 If A= $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ then find A^{-1} by using adjoint of the matrix.

Q.7 Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$

Q.8 Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

Q.9 Without expansion ,verify that $\begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix}=0$

Q.10 Solve the following systems of linear equations by Cramer's rule.

2x+2y+z=3

3x-2y-2z=1

5x+y-3z=2

Q.11 Use matrices to solve the following systems:

x - 2y + z = -1

3x + y - 2y = 4

y - z = 1

Q.12 Solve the system of linear equations by Cramer's rule.

$$3x_1 + x_2 - x_3 = -4$$

$$x_1 + x_2 - 2x_3 = -4$$

$$-x_1 + 2x_2 - x_3 = 1$$

Q.13 Use matrices to solve the system

$$x_1 - 2x_2 + x_3 = -4$$

$$2x_1 - 3x_2 + 2x_3 = -6$$

$$2x_1 + 2x_2 + x_3 = 5$$

Chapter 4:

Q.1 Solve by factorization $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$; $x \neq \frac{1}{a}, \frac{1}{b}$

Q.2 Solve by quadratic formula $(a+b)x^2 + (a + 2b + c)x + b + c = 0$

Q.3 Solve by quadratic formula

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

Q.4 Solve $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

Q.5 Solve $4.2^{2x+1} - 9.2^x + 1 = 0$

Q.6 Solve $3^{2x-1} - 12.3^x + 81 = 0$

Q.7 Show that $(1+\omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$ ….$2n$ factors=1

Q.8 Find the condition that one root of $ax^2 + bx + c = 0$,$a \neq 0$ is square of the other.

Q.9 $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

Q.10 If the roots of $px^2 + qx + q = 0$ are α and β, prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

Q.11 If α, β are the roots of $5x^2 - x - 2 = 0$ form the equation whose roots are $\frac{3}{\alpha}$ and $\frac{3}{\beta}$.

Q.12 Show that the roots of $x^2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2(1 + m^2)$.

Q.13 Show that the roots of $(mx + c)^2 = 4ax$ will be equal if $c = \frac{a}{m}; m \neq 0$

Q.14 Prove that will have equal roots if $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2}$=1 will have equal roots if $c^2 = a^2m^2 + b^2$; ; $a \neq 0, b \neq 0$

Q.15 Solve $3x + 4y = 25$; $\frac{3}{x} + \frac{4}{y} = 2$

Q.16 Solve the system of equation : $x + y = a + b$ and $\frac{a}{x} + \frac{b}{y} = 2$

Q.17 Prove that sum of three cube roots of unity is zero.

Q.18 Prove that $(-1 + \sqrt{-3})^4 + (-1 - \sqrt{-3})^4 = -16$

Q.19 If α, β are the roots of $5x^2 - x - 2 = 0$, form the equation whose roots are

$\frac{3}{\alpha}$ and $\frac{3}{\beta}$.

Q.20 Solve $x^2 + (y + 1)^2 = 18$; $(x + 2)^2 + y^2 = 21$

Chapter 6:

Q.1 Find n so that $\frac{a^n+b^n}{a^{n+1} + b^{n+1}}$ may be the A.M. between a and b.

Q.2 The sum of 9 terms of an A.P. is 171 and its eight term is 31. Find the series.

Q.3The sum of three numbers in an A.P. is 24 and their product is 440. Find the numbers.

Q.4 Find the four numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

Q.5 Find three consecutive numbers in G.P. whose sum is 26 and their product is 216.

Q.6 Show that the reciprocals of the terms of the terms of the geometric sequence

$a_{1,}$ $a_1 r^2$, $a_1 r^4$, ... from another geometric sequence.

Q.7 If the sum of the four consecutive terms in G.P. is 80 and A.M. of the second and the fourth of them is 30.Find the terms.

Q.8 If a,b,c,d are in G.P. prove that $a^2 - b^2, b^2 - c^2, c^2 - a^2$ are in G.P.

Q.9 For what value of n, is $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ the positive geometric mean between two distinct numbers a and b?

Q.10 If $y=\frac{x}{2}+\frac{1}{4}x^2+\frac{1}{8}x^3+\cdots$ and if $0< x < 2$, then prove that $x=\frac{2y}{1+y}$

Q.11 If $y=\frac{2}{3}x+\frac{4}{9}x^2+\frac{8}{27}x^3+\cdots$ and if $0< x < \frac{3}{2}$, then prove that $x=\frac{3y}{2(1+y)}$

Q.12 Find the five numbers in A.P. whose sum is 25 and sum of whose

Squares is 135.

Q.13 If S_2, S_3, S_5 are the sums of 2n,3n,5n terms of an A.P.,show that $S_5=5(S_3-S_2)$

Q.14 Show that the sum of n A.Ms. between a and b is equal to n times their A.M.

Q.15 If $y=1+\frac{x}{2}+\frac{x^2}{4}+\cdots$ then show that $x=2(\frac{y-1}{y})$

Q.16 The sum of an infinite geometric series is 9 and the sum of the squares of its

terms is $\frac{81}{5}$. Find the series.

Chapter 7:

Q.1Find the numbers greater than 23000 that can be formed from the digits 1,2,3,5,6

without repeating any digits.

Q.2 How many 6-digit numbers can be formed,without repeating any digit from the digits

0,1,2,3,4,5? In how many of them will 0 be at the tens place?

Q.3 Prove that ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r$

Q.4 How many 6-digit numbers can be formed from the digits 2,2,3,3,4,4? How many

Of them will lie between 400,000 and 430,000?

Q.5 In how many ways can be letters of the word MISSISSIPPI be arranged when all the

letters are to be used?

Q.6 Prove from the first principle that

i) ${}^nP_r=n.{}^{n-1}P_{r-1}$ ii) ${}^nP_r={}^{n-1}P_r+r.{}^{n-1}P_{r-1}$

Q.7 Find the value of n when

i) ${}^{n}P_{2}=30$ ii) ${}^{11}P_{n}=11.10.9$

Q.8 How many numbers greater than 1000,000 can be formed from the digits 0,2,2,2,3,4,4?

Q.9 Find the value of n and r , when ${}^{n}C_{r}=35$ and ${}^{n}P_{r}=210$

Chapter 8:

Q.1 Use mathematical induction to prove that

i) $1+3+5+\cdots+(2n-1)=n^2$

ii) $1+4+7+\cdots+(3n-2)=\frac{n(3n-1)}{2}$

iii) $1+5+9+\ldots+(4n-3)=n(2n-1)$

Q.2 Find the term independent of x in the expansion

i) $(\sqrt{x}+\frac{1}{2x^2})^{10}$ ii) $(x-\frac{2}{x})^{10}$

Q.3 Find the term involving x^4 in the expansion of $(3-2x)^7$

Q.4 Use binomial theorem to show that $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{2.4.6}+\cdots=\sqrt{2}$

Q.5 If x is so small that its square and higher powers can be neglected, then show that

$\frac{1-x}{\sqrt{1+x}}=1-\frac{3}{2}x$

Q.6 If $y=\frac{1}{3}+\frac{1.3}{2!}(\frac{1}{3})^2+\cdots$,then prove that $y^2+2y-2=0$

Q.7 Find the coefficient of x^5 in the expansion of $(x^2-\frac{3}{2x})^{10}$

Q.8 If x is very nearly equal to 1, then prove that $px^p-qx^q=(p-q)x^{p+q}$

Q.9 Find the term involving x^{-2} in the expression of $\left(x-\frac{2}{x^2}\right)^{13}$

Q.10 Determine the middle term or terms in the following expansions $\left(\frac{3}{2}x-\frac{1}{3x}\right)^{11}$

Q.11 If x is so small that its square and higher powers can be neglected, then show that

$\frac{1+x}{\sqrt{1-x}} = 1 + \frac{3}{2}x$

Q.12 If $2y = \frac{1}{2^2} + \frac{1.3}{2!}\frac{1}{2^4} + \frac{1.3.5}{3!}\frac{1}{2^6} + \cdots$, then prove that $4y^2 + 4y - 1 = 0$

Chapter 9

Q.1 If $cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in first quadrant, find the value $cos\theta\ and\ cosec\theta.$

Q.2 If $cosec\theta = \frac{m^2+1}{2m}$ and 0< $\theta < \frac{\pi}{2}$ find the value of the remaining trigonometric ratio.

Q.3 Prove the identity $\frac{1}{cosec\theta - cot\theta} - \frac{1}{sin\theta} = \frac{1}{\sin\theta} - \frac{1}{cosec\theta + cot\theta}$

Q.4 Prove that $\frac{tan\theta + sec\theta - 1}{tan\theta + sec\theta + 1} = tan\theta + sec\theta$

Q.5 Prove that $\frac{1 - sin\theta}{cos\theta} = \frac{cos\theta}{1 + sin\theta}$

Q.6 $sin^6\theta - cos^6\theta = (sin^2\theta - cos^2\theta)(1 - sin^2\theta cos^2\theta)$

Q.7 $sin^6\theta + cos^6\theta = 1 - 3\ sin^2\theta cos^2\theta$

Q.8 If $tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in the III quadrant,

find the value of $\frac{csc^2\theta - sec^2\theta}{csc^2\theta + sec^2\theta} = \frac{3}{4}$.

Q.9 Find the value of the other five trigonometric functions of θ, if $cos\theta = \frac{12}{13}$ and the terminal side of the angle is not in the I quadrant.

Q.10 Prove that $(tan\theta + cot\theta)^2 = sec^2\theta csc^2\theta$

Q.11 If $cot\theta = \frac{5}{2}$ and terminal arm of the angle is in the first quadrant, find the value of $\frac{3sin\theta + 4cos\theta}{cos\theta - sin\theta}$.

Chapter 10

Q.1 Prove the identity $\frac{sin3\theta}{sin\theta} - \frac{cos3\theta}{cos\theta} = 2$

Q.2 If $sin\alpha = \frac{4}{5}$ and $= \frac{40}{41}$, where 0< $\alpha < \frac{\pi}{2}$ and 0< $\beta < \frac{\pi}{2}$ Show that sin$(\alpha - \beta)$=$\frac{113}{205}$

Q.3 Reduce $cos^4\theta$ to an expression involving only function of multiple of θ, raised to the first power.

Q.4 $\sqrt{\frac{1+sin\alpha}{1-sin\alpha}} = \frac{sin\frac{\alpha}{2}+cos\frac{\alpha}{2}}{sin\frac{\alpha}{2}-cos\frac{\alpha}{2}}$

Q.5 Show that $\cot(\alpha + \beta) = \frac{cot\alpha cot\beta - 1}{cot\alpha + cot\beta}$

Q.6 Reduce $sin^4\theta$ to an expression involving only function of multiple of θ, raised to the first power.

Q.7 Prove that $\frac{sin\theta+sin3\theta+sin5\theta+sin7\theta}{cos\theta+cos3\theta+cos5\theta+cos7\theta} = tan4\theta$

Q.8 Prove that $cos20°cos40°cos60°cos80° = \frac{1}{16}$

Q.9 Prove that $sin\frac{\pi}{9}sin\frac{2\pi}{9}sin\frac{\pi}{3}sin\frac{4\pi}{9} = \frac{3}{16}$

Q.10 Prove that $sin10°sin30°sin50°sin70° = \frac{1}{16}$

Q.11 Prove that $\frac{cos8°-sin8°}{cos8°+sin8°} = tan37°$

Q.12 If α, β, γ are the angles of triangle ABC ,show that

$$cot\frac{\alpha}{2} + cot\frac{\beta}{2} + cot\frac{\gamma}{2} = cot\frac{\alpha}{2}cot\frac{\beta}{2}cot\frac{\gamma}{2}$$

Q.13 Prove that $\frac{sinA+sin2A}{1+cosA+cos2A} = tanA$

Q.14 Show that $\cos(\alpha+\beta)\cos(\alpha-\beta) = cos^2\alpha - sin^2\beta = cos^2\beta - sin^2\alpha$

Chapter 12

Q.1 Solve the triangle ABC, in which $a = 3, c = 6, \beta = 36°20'$

Q.2 Solve the triangle ABC, in which $a = 7, b = 3, \gamma = 38°13'$

Q.3 Solve the triangle ABC, in which $a = 32, b = 40, c = 66$

Q.4 The sides of triangle are $x^2 + x + 1, 2x + 1\ and\ x^2 - 1$. Prove that the greatest angle of the triangle is 120°.

Q.5 Show that $r = asin\frac{\beta}{2}sin\frac{\gamma}{2}\ sec\frac{\alpha}{2}$

Q.6 Show that $r_1 = 4Rsin\frac{\alpha}{2}cos\frac{\beta}{2}cos\frac{\gamma}{2}$

Q.7 Show that $r_1 = stan\frac{\alpha}{2}$

Q.8 Prove that in equilateral triangle $r : R : r_1 = 1 : 2 : 3$

Q.9 Prove that $r = stan\frac{\alpha}{2}\ tan\frac{\beta}{2}tan\frac{\gamma}{2}$

Q.10 Prove that $(r_1 + r_2)tan\frac{\gamma}{2} = c$

Q.11 With usual notations, prove that $R = \frac{abc}{4\Delta}$

Q.12 With usual notations, prove that $r = \frac{\Delta}{s}$

Q.13 Prove that Law of Cosine.

Q.14 Prove that Law of Sine.

Q.15 Show that $r = (s - a)tan\frac{\alpha}{2} = (s - b)tan\frac{\beta}{2} = (s - c)tan\frac{\gamma}{2}$

Q.16 Prove that $abc(sin\alpha + sin\beta + sin\gamma) = 4\Delta s$.

Q.17 Prove that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

Q.18 Prove that $r_1 + r_2 + r_3 - r = 4R$

Chapter 13

Q.1 Prove that $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=\cos^{-1}\frac{253}{325}$

Q.2 prove that $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}$

Q.3 Prove that $\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\frac{\pi}{4}$

Q.4 Prove that $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=\frac{\pi}{4}$

Q.5 Prove that $\sin^{-1}A+\sin^{-1}B=\sin^{-1}(A\sqrt{1-B^2}+B\sqrt{1-A^2}$

Q.6 Prove that $\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{15}{17}$

Q.7 Prove that $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

Q.8 Prove that $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$