



Forman Christian College, Lahore (A Chartered University)

Formanite Physics Notes

Part I

Physics Department

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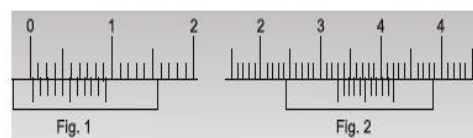
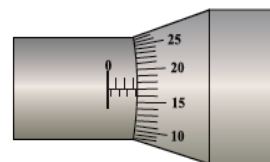
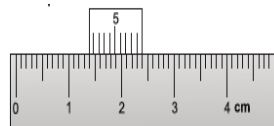
Chapter 1

Exercise

Measurements

MCQs

- 1.1 The purpose of study and discoveries in physics is:
 (A) The probing of interstellar spaces (B) The betterment of mankind
 (C) The development of destructive technology in warfare
 (D) Development in aesthetics for the world
- 1.2 The length of steel pipe is in between 0.7 m to 0.8 m. Identify from the following, the appropriate instrument to be used for an accuracy of 0.001 m:
 (A) A micrometer screw gauge (B) A meter rule
 (C) A ten meters measuring tap (D) A Vernier Callipers
- 1.3 The diameter of a steel ball is measured using a Vernier Calipers and its reading is shown in the figure. What is the diameter of the steel ball?
 (A) 1.30 cm (B) 1.39 cm (C) 1.40 cm
 (D) 1.31 cm
- 1.4 The figure shows the reading on a micrometer screw gauge used to measure the diameter of a thin rod. One complete turn of the thimble is 0.50 mm and there are 50 lines on the circular scale. The diameter of the rod is:
 (A) 3.67 mm (B) 3.17 mm (C) 4.17 mm
 (D) 4.20 mm
- 1.5 The number of significant figures of a measurement are defined as:
 (A) they reflect the accuracy of the observation in a measurement
 (B) they are the figures which are reasonably reliable
 (C) they are the accurately known digits and the first doubtful digit of a measurement
 (D) All of the above
- 1.6 The number of the significant figures in the measured mass 2500.0 kg is:
 (A) two (B) three (C) four (D) five
- 1.7 The sum of 12 kg + 2.02 kg + 5.1 kg according to appropriate precision is:
 (A) 19 kg (B) 19.0 kg (C) 19.1 kg (D) 19.12 kg
- 1.8 The answer to appropriate precision for the subtraction (1.126 – 0.97268) is:
 (A) 0.15 (B) 0.153 (C) 0.1533 (D) 0.15332
- 1.9 The answer to the product (2.8723 × 1.6) to the appropriate number of significant figures is:
 (A) 4.59568 (B) 4.595 (C) 4.59 (D) 4.6
- 1.10 The answer to the mathematical division (45.2 ÷ 6.0) in appropriate number of significant figures is:
 (A) 7.5 (B) 7.53 (C) 7.533 (D) 7.5333
- 1.11 The answer to the following mathematical operation $\frac{24.4 \text{ m} \times 100 \text{ m}}{5.0 \text{ m}}$ to the appropriate number of significant figures is:
 (A) 4880 m (B) 4900 m (C) $4.88 \times 10^3 \text{ m}$ (D) $4.9 \times 10^2 \text{ m}$
- 1.12 The ratio of the dimensions of force and energy is:
 (A) T (B) T^{-1} (C) L (D) L^{-1}
- 1.13 Identify which pair from the following does not have identical dimensions.
 (A) Work and torque (B) Angular momentum and Plank's constant
 (C) Moment of inertia and moment of force (D) Impulse and momentum
- 1.14 The following figures are of the same Vernier callipers. Figure (1) shows the reading when the jaws are closed while figure (2) shows the reading when a solid cylinder is placed between the jaws. The length of the cylinder is:
 (A) 3.26 cm (B) 3.30 cm (C) 3.34 cm (D) 4.20 cm



- 1.15 The least count of an instrument determines:**
 (A) precision of a measurement (B) accuracy of a measurement
 (C) fractional uncertainty of a measurement (D) percentage uncertainty of a measurement
- 1.16 A measuring tape has been graduated with a minimum scale division of 0.2 cm. The allowed reading using this tape may be:**
 (A) 80.5 cm (B) 80.6 cm (C) 80.65 cm (D) 80.7 cm

Answer Key							
Q.No.	Option	Q.No.	Option	Q.No.	Option	Q.No.	Option
1.1	B	1.5	D	1.9	D	1.13	C
1.2	B	1.6	D	1.10	A	1.14	A
1.3	B	1.7	A	1.11	D	1.15	A
1.4	A	1.8	B	1.12	D	1.16	B

Short Answer Questions

- 1.1 What are base units and derived units? Give some examples of both these units.**

Ans Base Units: The units of base quantities are called base units. There are seven base units namely meter, kilogram, second, ampere, kelvin, candela and mole.

Derived Units: The units which are derived from base units are called derived units. Some of the derived units are newton, watt, joule, etc.

- 1.2 How many significant figures should be retained in the following?**

Ans i) Multiplying or dividing several numbers: In Multiplication and division, the factor containing the least number of significant figures should be retained.

ii) Adding or subtracting numbers: In adding or subtracting numbers, the smallest number of decimal places in any of the quantities being added or subtracted should be retained.

- 1.3 How is the Vernier scale related to the main scale of a Vernier Callipers? What is meant by L.C of the Vernier Callipers?**

Ans Least Count: The smallest measurement which can be taken by the Vernier Callipers is known as the least count.

Relation:

10 Vernier divisions = 9 divisions of main scale

1 Vernier division = 0.9 main scale divisions

Least Count of Vernier Callipers = 1 - 0.9 mm = 0.1 mm

- 1.4 Write the following numbers in scientific notation:**

Ans a) 143.7 = 1.437 × 10² b) 206.4 × 10² = 2.064 × 10⁴

- 1.5 Write the following numbers using correct prefixes.**

Ans a) 580 × 10² g = 58 × 10³ g = 58 kg b) 0.45 × 10⁻⁵ s = 4.5 × 10⁻⁶ s = 4.5 μs

- 1.6 Kinetic energy of a body of mass m moving with speed v is given by $\frac{1}{2}mv^2$. What are the dimensions of kinetic energy?**

Ans K.E. = $\frac{1}{2}mv^2$

Dimensions of K.E. = [M][L²T⁻²] = [M][L²T⁻²] = [M L²T⁻²]

- 1.7 How many significant figures are there in the following measurements?**

Ans a) 37 km = 2 b) 0.002953 m = 4 c) 7.50034 cm = 6 d) 200.0 m = 4

- 1.8 Write the dimensions of:**

i) Planck's constant

Ans E = hf or h = E/f

Dimensions of h = [M L²T⁻²] / [T⁻¹] = [M L²T⁻¹]

ii) Angular Velocity

v = r ω or ω = $\frac{v}{r}$

Dimensions of ω = $\frac{[LT^{-1}]}{[L]} = [L^{1-1} T^{-1}] = [T^{-1}]$

Constructed Response Questions

1.1 Why do we find it useful to have two units for the amount of a substance, the kilogram and the mole?

Ans **Kilogram** measures the quantity of a substance having relatively large mass. One kg has a different number of atoms / molecules in every substance.

Mole is used to count the number of atoms/molecules in a substance. One mole of every substance has same number of atoms/molecules i.e. 6.02×10^{23} atoms/molecules.

1.2 Three students measured the length of a rod with a scale on which minimum division is 1 mm and recorded as i) 0.4235 m (ii) 0.42 m (iii) 0.424 m. Which record is correct and why?

Ans 1 mm = 1 / 1000 m = 0.001 m.

(iii) 0.424 m is correct because the least count of meter rod is up to 3 decimal places.

1.3 Why is the kilogram (not the gram), the base unit of mass.

Ans The kilogram was chosen as a base unit of mass because it is practically convenient unit for everyday usage of large-scale measurements. It provides a manageable scale for most measurements.

1.4 Consider the equation: $P = Q + R$. If Q and R both have the dimensions of [MLT], what are the dimensions of P? What are the units of P in SI? If the dimensions of Q were different from those of R, could we determine dimensions of P?

Ans a) As $P = Q + R$,

$$[P] = [Q] + [R]$$

$$[P] = [MLT] + [MLT] = 2[MLT]$$

As 2 is dimensionless, So

$$[P] = [MLT]$$

b) SI units of $P = \text{kg m s}$

c) No, it cannot be determined as dimensions of different quantities cannot be added or subtracted.

1.5 What is the least count of a clock if it has:

a) Hour's hand, minute's hand and second's hand

Ans Smallest time interval measured by the clock is one second. Therefore, least count of clock is 1 second.

b) Hour's hand and minute's hand

Ans Smallest time interval measured by the clock is one minute. Therefore, least count of clock is 1 minute.

1.6 How can the diameter of a round pencil be measured using meter rule with the same accuracy as that of Vernier Callipers? Describe.

Ans Least count of meter rule = 0.1 cm

Least count of Vernier Callipers = 0.01 cm.

- Arrange 10 identical round pencils tightly side-by-side.
- Use the meter rule to measure their combined width carefully.
- Divide the above width by total number of pencils to get average diameter.
- This average effectively increases accuracy to match the Vernier Calipers accuracy.

1.7 How would be the readings differ if the screw gauge is used instead of a Vernier Callipers to measure the thickness of a glass plate?

Ans Least count of screw gauge = 0.001 cm

Least count of Vernier Callipers = 0.01 cm.

The measurement taken by screw gauge will be more precise than the Vernier Callipers.

1.8 Write the correct reading of the length of a solid cylinder as shown in the figure if there is an error of + 0.02 cm in the Vernier Callipers.

Ans Main Scale Reading = 2.6 cm

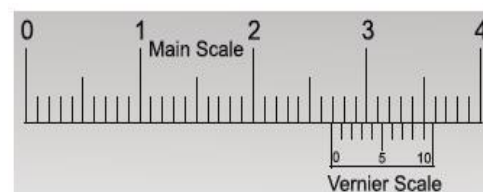
No. of vernier scale coinciding with main scale = 9

Vernier Scale Reading = $9 \times 0.01 \text{ cm} = 0.09 \text{ cm}$

Observed reading = $2.6 + 0.09 \text{ cm} = 2.69 \text{ cm}$

Zero correction = - 0.02 cm

Corrected value = $2.69 - 0.02 \text{ cm} = 2.67 \text{ cm}$



1.9 There are 50 divisions on the circular scale of a screw gauge. If the head (thimble) of the screw is given 10 revolutions, then the spindle advances by 5mm. There is also zero as the 2nd division of the circular scale coincides with the datum line and zero of circular scale is below the datum line. What is the thickness of a glass slab as measured by the described screw gauge shown in the figure?

Ans Main scale reading = 6.5 mm

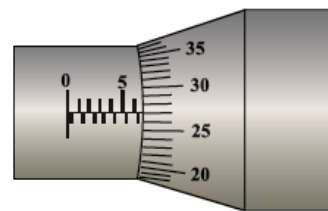
No. of circular scale division coinciding with datum line = 27

Circular scale reading = $27 \times 0.01 \text{ mm} = 0.27 \text{ mm}$

Observed value = $6.5 + 0.27 = 6.77 \text{ mm}$

Zero correction = $- 0.02 \text{ mm}$

Corrected value = $6.77 - 0.02 = 6.75 \text{ mm}$



1.10 What is meant by a dimensionless quantity? Give one example.

Ans A physical quantity that has no dimension is called dimensionless quantity. It has numerical value only.

Example: refractive index, strain etc.

1.11 A student uses a screw gauge to determine the thickness of a sheet of paper. The student folds the paper three times and measures the total thickness of the folded sheet. Assume that there is no zero error in the screw gauge. The reading of screw gauge is shown in the figure. Find the thickness of the sheet.

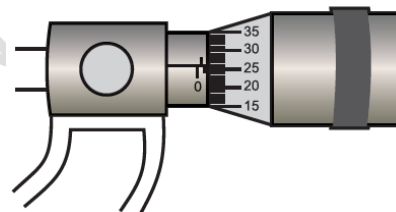
Ans Main scale reading of screw gauge = 1 mm

No. of circular scale division coinciding with datum line = 26

Circular scale reading = $26 \times 0.01 \text{ mm} = 0.26 \text{ mm}$

Observed value = $1 + 0.26 = 1.26 \text{ mm}$

Thickness of sheet = $1.26/8 = 0.16 \text{ mm}$



1.12 Round off each of the following numbers to 3 significant figures and write your answer in scientific notation.

Ans (a) $0.02055 = 0.0206 = 2.06 \times 10^{-2}$

(b) $4656.5 = 4.6565 \times 10^3 = 4.66 \times 10^3$

Comprehensive Questions

1.1 What is meant by uncertainty in a measurement? How the uncertainty in a digital instrument is indicated?

Ans **Uncertainty in Measurement:**

Uncertainty is the doubt or inaccuracy in a measurement due to the limitation of the instrument. It reflects the range within which the true value lies.

The maximum uncertainty can be estimated as one smallest division of the instrument. This is called the absolute uncertainty (least count) of the instrument.

Example: The absolute uncertainty of a meter rule is 1 mm.

Explanation:

Every instrument is calibrated to a certain smallest division marked on it. This puts a limit to its accuracy. When you take a reading with an instrument, its limit of measurement is the smallest division on its scale. Hence, every measured quantity has some uncertainty about its value.

When a measurement is made, it is taken to the nearest marking on the scale.

For example, if one edge of the book coincides with 10.0 cm mark and the other with 33.5 cm, then length with uncertainty = $(33.5 \pm 0.05) \text{ cm} - (10.0 \pm 0.05) \text{ cm} = (23.5 \pm 0.1) \text{ cm}$

It means that the true length of the book is in between 23.4 cm and 23.6 cm. Hence, the maximum uncertainty is $\pm 0.05 \text{ cm}$, which is equivalent to an uncertainty of 0.1 cm. In fact, it is equal to least count of the meter rule.

Uncertainty may be recorded as:

Fractional uncertainty = $\frac{\text{Absolute uncertainty}}{\text{Measured value}}$

or

Percentage uncertainty = $\frac{\text{Absolute uncertainty}}{\text{Measured value}} \times 100$

Uncertainty in Digital Instruments

Some modern instruments use a digital scale that give precise readings, but they still have some uncertainty. This uncertainty can be reduced by the following methods:

1. With a digital scale, we usually estimate one digit beyond the certain digit. This causes some fluctuations of the last digit. If the last digit fluctuates by 1 or 2, we down that last digit. If fluctuation in the last digit is more than 2, it may mean that the reading is being influenced by some factor such as air currents. A large fluctuation may mean that the displayed digit is not really significant.
2. The indication of uncertainty in a recorded measurement can be simplified using significant figures. If a measurement is recorded using significant figures, then the last estimated digit, is an indication of the accuracy of the recorded value.

1.2 Differentiate between the terms precision and accuracy with reference to measurement of physical quantities.

Ans The terms precision and accuracy are frequently used in physics measurements.

Precision:

A precise measurement is the one which has less precision or absolute uncertainty.

The precision of a measurement is determined by the instrument or device being used. The smaller the least count the more precise is the measurement.

Accuracy:

An accurate measurement is the one which has less fractional or percentage uncertainty.

Accuracy is defined as the closeness of a measurement to the exact or accepted value of a physical quantity. It is expressed by the fractional or percentage uncertainty. The smaller the fractional or percentage uncertainty, the more accurate is the measurement.

Explanation:

Example-1: The length of an object is recorded as 25.5 cm by using a meter rule having smallest division in millimeter. Its precision or absolute uncertainty (least count) = ± 0.1 cm.

$$\text{Fractional uncertainty} = \frac{0.1 \text{ cm}}{25.5 \text{ cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1 \text{ cm}}{25.5 \text{ cm}} \times 100\% = 0.4 \%$$

Example-2: Another measurement taken by Vernier Callipers with least count 0.01 cm is recorded as 0.45 cm. It has precision or absolute uncertainty (least count) = ± 0.01 cm.

$$\text{Fractional uncertainty} = \frac{0.1 \text{ cm}}{0.45 \text{ cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.1 \text{ cm}}{0.45 \text{ cm}} \times 100\% = 2 \%$$

Conclusion:

Thus, the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by Vernier Callipers is more precise but is less accurate.

In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometer screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that:

We can never make an exact measurement. The best we can do is to come as close as possible within the limitation of the measuring instrument.

1.3 (a)What is meant by significant figures? Write two reasons for using them in measurements. How to find the uncertainty in a timing experiment such as the time period of a simple pendulum?

Ans Significant Figures: In any measurement, the accurately known digits and the first estimated or doubtful digit are called significant figures.

Reasons:

1. Proper use of significant figures ensures that we correctly represent the uncertainty of our measurements.

2. As the quality of measuring instruments is improved, we extend the result to more significant figures and correspondingly improve the experimental accuracy of the result.

The uncertainty in a timing experiment:

The uncertainty in the time period of a vibrating body is found by dividing the least count of the timing device by the number of vibrations.

For example, the time of 30 vibrations of a simple pendulum recorded by a stopwatch accurate up to one tenth of a second is 54.6 s, the period

$$T = \frac{54.6 \text{ s}}{30} = 1.82 \text{ s with uncertainty } \frac{0.1 \text{ s}}{30} = 0.003 \text{ s}$$

Thus, period T is quoted as $T = 1.82 \pm 0.003 \text{ s}$.

Hence, it is advisable to count a large number of swings to reduce timing uncertainty.

(b) The mass of a solid cylinder is 12.85 g. Its length is 3.35 cm and diameter is 1.25 cm. Find the density of its material expressing the uncertainty in the density.

Ans Given data;

$$m = 12.85 \text{ g} \pm 0.01 \text{ g}$$

$$l = 3.35 \text{ cm} \pm 0.01 \text{ cm}$$

$$d = 1.25 \text{ cm} \pm 0.01 \text{ cm}$$

To find: density with uncertainty =?

Solution: As we know

$$r = \frac{d}{2} = \frac{1.25}{2} = 0.625 \text{ cm}$$

Also, Volume of cylinder is

$$V = \pi r^2 l = 3.14 \times (0.625)^2 \times 3.35 = 4.109 \text{ cm}^3$$

Density is given by

$$\rho = \frac{m}{V} = \frac{12.85}{4.109} = 3.13 \text{ g cm}^{-3}$$

Percentage uncertainty in density is given by

$$\% \text{ uncertainty in } \rho = \% \text{ uncertainty in } m + \% \text{ uncertainty in } V \text{ -----(1)}$$

$$\text{Here } \% \text{ uncertainty in } m = \frac{0.01}{12.85} \times 100 = 0.08 \%$$

$$\% \text{ uncertainty in } V = 2 (\% \text{ uncertainty in } r) + (\% \text{ uncertainty in } l)$$

$$= 2 \left(\frac{0.005}{0.625} \times 100 \right) + \left(\frac{0.01}{3.35} \times 100 \right)$$

$$= 2 (0.8 \%) + (0.3 \%)$$

$$= 1.9 \%$$

Putting values in (1),

$$\% \text{ uncertainty in } \rho = 0.08 \% + 1.9 \% = 1.98 \%$$

Thus, $\rho = 3.13 \text{ g cm}^{-3}$ with 1.98 % uncertainty

$$\text{i.e., } \rho = 3.13 \pm 0.06 \text{ g cm}^{-3}$$

where 3.13 g cm^{-3} is the calculated density and 0.06 g cm^{-3} is the uncertainty in it.

1.4 Explain with examples the writing of physical quantities into their dimensions. Write its two benefits.

Ans Dimension: A dimension is a measurable property of a physical quantity that describes its qualitative nature in terms of fundamental quantities such as mass, length, time, temperature, and electric current.

It is denoted by a symbol written in square brackets.

Explanation:

Each base quantity is considered a dimension. The quantities such as length, depth, height, diameter, light year are all measured in metre and are denoted by the same dimension [L].

Similarly, the other fundamental quantities, mass, time, electric current and temperature are denoted by specific symbols [M], [T], [A] and [θ], respectively.

These five dimensions have been chosen as being basic because they are easy to measure in experiments. The dimensions of other quantities indicate how they are related to the basic quantities and are combinations of fundamental dimensions.

Examples:

Speed v is measured in meters per second, so it has the dimensions

$$[v] = \frac{[L]}{[T]} = [L][T]^{-1} = [LT^{-1}]$$

As the acceleration $a = \frac{\Delta v}{\Delta t}$

Dimensions of acceleration are:

$$[a] = \frac{[v]}{[T]} = \frac{[LT^{-1}]}{[T]} = [LT^{-1}][T]^{-1} = [LT^{-2}]$$

Uses of Dimensional Analysis:

By the use of dimensionality, we can check the homogeneity (correctness) of a physical equation, and also, we can derive a formula for a physical quantity.

1. Homogeneity of Physical Equation

The correctness of an equation can be checked by showing that the dimensions of quantities on both sides of the equation are the same. This is known as the principle of homogeneity.

Suppose a car starts from rest ($v_i=0$) and covers a distance S in time t moving with an acceleration a . The equation of motion is given by:

$$S = v_i t + \frac{1}{2} at^2$$

or

$$S = \frac{1}{2} at^2 \quad (\because v_i=0)$$

Numerical factors like $\frac{1}{2}$ have no dimensions, so they can be ignored. By putting the dimensions of both sides of the equation:

$$[S] = [a][t^2]$$

Writing the symbols of dimensions:

$$[L] = [LT^{-2}][T^2]$$

$$[L] = [LT^{-2}T^2]$$

$$[L] = [L]$$

This shows that dimensions on both sides of the equation are the same; therefore, the equation is dimensionally correct.

2. Deriving a Possible Formula

Dimensionality can be used to derive a possible formula for a physical quantity by correct estimation of various factors on which the quantity depends.

1.5 Check the homogeneity of the relation:

$$v = \sqrt{\frac{T \times \ell}{m}}$$

where v is the speed of transverse wave on a stretched string of tension T , length ℓ and mass m .

Solution:

Dimensions of L.H.S

$$[v] = [LT^{-1}] \text{ -----(1)}$$

Dimensions of R.H.S

$$\begin{aligned} &= ([T] \times [l] \times [m^{-1}])^{\frac{1}{2}} \\ &= ([MLT^{-2}] \times [L] \times [M^{-1}])^{\frac{1}{2}} \quad (\because T \text{ is a force}) \\ &= ([ML^2T^{-2} \times M^{-1}])^{\frac{1}{2}} \\ &= [LT^{-1}] \text{ -----(2)} \end{aligned}$$

From (1) and (2), since dimensions of both sides of the equation are the same, hence the given equation is dimensionally correct.

Numerical Problems

1.1 Astronomers usually measure astronomical distances in light years. One light year is the distance that light travels in one year. If speed of light is $3 \times 10^8 \text{ m s}^{-1}$, what is one light year in meters?

Given data:

$$v = c = 3.0 \times 10^8 \text{ m s}^{-1}$$

$$t = 1 \text{ year} = 365 \times 24 \times 60 \times 60 \text{ s}$$

$$= 31536000 \text{ s}$$

To find:

$$\text{Distance} = S = ?$$

Solution:

$$S = c \times t$$

$$= 3.0 \times 10^8 \times 31536000$$

$$S = 9.5 \times 10^{15} \text{ m} \quad \text{Ans}$$

1.2 Write the estimated answer of the following in the standard form.

- (a) How many seconds are there in 1 year?
 (b) How many years are in 1 second?

Given data:

(a) $t = 1 \text{ year}$

To find:

$$\text{Number of seconds in one year} = ?$$

Solution:

$$t = 1 \text{ year}$$

$$= 1 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$= 31536000 \text{ s}$$

$$t = 3.2 \times 10^7 \text{ s} \quad \text{Ans}$$

Given data:

(b) $t = 1 \text{ year}$

To find:

$$\text{Years in 1 second} = ?$$

Solution:

$$t = 1 \text{ year}$$

$$1 \text{ year} = 1 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

$$\left(\frac{1}{1 \times 365 \times 24 \times 60 \times 60} \right) \text{ yr} = 1 \text{ s}$$

$$1 \text{ s} = 3.1 \times 10^{-8} \text{ years} \quad \text{Ans}$$

1.3 The length and width of a rectangular plate are measured to be 18.3 cm and 14.60 cm, respectively. Find the area of the plate and state the answer to correct number of significant figures.

Given data:

$$l = 18.3 \text{ cm}$$

To find:

$$\text{Area} = A = ?$$

Solution:

$$A = l \times w$$

$$= 18.3 \times 14.60$$

$$= 267.18 \text{ cm}^2$$

$$A = 267 \text{ cm}^2 \quad \text{Ans}$$

1.4 Find the sum of the masses given in kg up to appropriate precision:

- (i) 3.197 (ii) 0.068 (iii) 13.9 (iv) 3.28

Given data:

$$m_1 = 3.197 \text{ kg}$$

$$m_2 = 0.068 \text{ kg}$$

$$m_3 = 13.9 \text{ kg}$$

$$m_4 = 3.28 \text{ kg}$$

To find:

$$\text{Total mass} = m = ?$$

Solution:

$$\begin{aligned}m &= m_1 + m_2 + m_3 + m_4 \\&= 3.197 + 0.068 + 13.9 + 3.28 \\&= 20.445 \text{ kg} \\m &= \mathbf{20.4 \text{ kg}} \quad \text{Ans}\end{aligned}$$

1.5 The diameter and length of a metal cylinder measured with the help of a Vernier Callipers of least count 0.01 cm are 1.22 cm and 5.35 cm respectively. Calculate its volume and uncertainty in it.

Given data:

$$\begin{aligned}\text{Diameter} &= d = 1.22 \text{ cm with least count } 0.01 \text{ cm} \\ \text{Length} &= l = 5.35 \text{ cm with least count } 0.01 \text{ cm} \\ \text{Absolute uncertainty in length} &= 0.01 \text{ cm}\end{aligned}$$

To find:

$$\text{Volume } V \text{ of the cylinder and uncertainty} = ?$$

Solution:

$$\text{Percentage uncertainty in length} = \frac{0.01 \text{ cm}}{5.35 \text{ cm}} \times 100 = 0.2 \%$$

$$\text{Absolute uncertainty in diameter} = 0.01 \text{ cm}$$

$$\text{Percentage uncertainty in diameter} = \frac{0.01 \text{ cm}}{1.22 \text{ cm}} \times 100 = 0.8 \%$$

$$\text{As volume is} \quad V = \frac{\pi d^2 l}{4}$$

$$\begin{aligned}\text{Total uncertainty in } V &= 2 (\% \text{age uncertainty in diameter}) + (\% \text{age uncertainty in length}) \\&= 2 \times 0.8 + 0.2 = 1.8 \%\end{aligned}$$

Then

$$\begin{aligned}V &= \frac{3.14 \times (1.22 \text{ cm})^2 \times 5.35 \text{ cm}}{4} \\&= 6.2509079 \text{ cm}^3 \text{ with } 1.8\% \text{ uncertainty} \\V &= (6.2 \pm 0.1) \text{ cm}^3\end{aligned}$$

Thus

where 6.2 cm^3 is calculated volume and 0.1 cm^3 is the uncertainty in it.

1.6 Show that the expression; $v_f^2 - v_i^2 = 2aS$ is dimensionally correct, where v_i is the initial velocity, a is the acceleration and v_f is the velocity after covering a distance S .

Solution:

Dimensions of R.H.S

$$\begin{aligned}&= [LT^{-1}]^2 - [LT^{-1}]^2 \\&= [L^2T^{-2}] - [L^2T^{-2}]\end{aligned}$$

Dimensions of L.H.S

$$\begin{aligned}&= 2 [LT^{-2}] [L] \\&= 2 [L^2T^{-2}]\end{aligned}$$

Hence

$$\text{Dimensions of L. H. S} = \text{Dimensions of R. H. S.}$$

1.7 Show that the famous “Einstein equation” $E = mc^2$ is dimensionally consistent.

Solution: $[E] = [m][c^2]$

$$\begin{aligned}\text{Dimensions of L.H.S} &= [E] \\&= [ML^2T^{-2}]\end{aligned}$$

$$\begin{aligned}\text{Dimensions of L.H.S} &= [m][c^2] \\&= [M][L^2T^{-2}]\end{aligned}$$

$$\text{As} \quad \text{Dimensions of L.H.S} = \text{Dimensions of R.H.S}$$

So, the equation is dimensionally correct.

1.8 Derive a formula for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period T may depend are:

- (i) length of the pendulum l
- (ii) mass of the bob m
- (iii) angle θ which the thread makes with the vertical

(iv) accelerates due to gravity g .

Solution:

$$T \propto l^a m^b \theta^c g^d$$

$$T = \text{constant } l^a m^b \theta^c g^d \text{ --- (1)}$$

Putting dimensions on both sides

$$[T] = \text{Constant} [L]^a [M]^b [LL^{-1}]^c [LT^{-2}]^d$$

Comparing dimensional power of $[T]$

$$1 = -2d$$

$$d = -\frac{1}{2}$$

Comparing dimensional power of $[M]$

$$0 = b$$

Comparing dimensional power of $[L]$

$$0 = a + c - c + d$$

$$d = -a$$

$$a = \frac{1}{2}$$

From equation 1,

$$T = \text{Constant } l^{\frac{1}{2}} m^0 \theta^0 g^{-\frac{1}{2}}$$

$$T = \text{Constant } \left(\frac{l}{g}\right)^{\frac{1}{2}}$$

$$T = \text{Constant } \sqrt{\frac{l}{g}}$$

Chapter 2

Exercise

Force and Motion

MCQs

- 2.1 The angle at which dot product becomes equal to cross product:
(A) 65° (B) 45° (C) 76° (D) 30°
- 2.2 The projectile gains its maximum height at an angle of:
(A) 0° (B) 45° (C) 60° (D) 90°
- 2.3 The scalar product of two vectors is maximum if they are:
(A) perpendicular (B) parallel (C) at 30° (D) at 45°
- 2.4 The range of projectile is same for two angles which are mutually:
(A) perpendicular (B) supplementary (C) complementary (D) 270°
- 2.5 The acceleration at the top of a trajectory of projectile is:
(A) maximum (B) minimum (C) zero (D) g
- 2.6 SI unit of impulse is:
(A) kg m s^{-2} (B) N m (C) N s (D) N m^2
- 2.7 The rate of change of momentum is:
(A) force (B) impulse (C) acceleration (D) power
- 2.8 As rocket moves upward during its journey, then its acceleration goes on:
(A) increasing (B) decreasing (C) remains same (D) it moves with uniform velocity
- 2.9 Elastic collision involves:
(A) loss of energy (B) gain of energy
(C) no gain, no loss of energy (D) no relation between energy & elastic collision

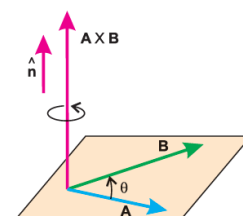
Answer Key					
Q.No.	Option	Q.No.	Option	Q.No.	Option
2.1	B	2.4	C	2.7	A
2.2	D	2.5	D	2.8	A
2.3	B	2.6	C	2.9	C

Short Answers Questions

- 2.1 State right hand rule for two vectors with reference to vector product.

Ans **Right-Hand-Rule:**

- Join the tail of vectors **A** and **B** which defines the plane of vectors.
- Rotate the first vector **A** into **B** through the smaller of two possible angles.
- Curl the fingers of the right hand in the direction of rotation, keeping the thumb erect.
- The direction of the product vector will be along the erect thumb i.e. perpendicular to the plane.



- 2.2 Define impulse and show how it is related to momentum.

Ans **Impulse:** The product of average force F that acts during time t is called Impulse. It is given by

$$\vec{I} = \vec{F} \times t$$

Units of Impulse = N s

Relationship between Impulse and Momentum:

We know that $\vec{I} = \vec{F} \times t$

From Newton's 2nd law in terms of momentum we have

$$\vec{F} = \frac{\Delta \vec{p}}{t}$$

Therefore, $\vec{I} = \frac{\Delta \vec{p}}{t} \times t$

$$\vec{I} = \Delta \vec{p} = m\vec{v}_f - m\vec{v}_i$$

2.3 Differentiate between an elastic & an inelastic collision.

Ans Elastic collision: The collision in which kinetic energy of the system is conserved, is called elastic collision.

Inelastic collision: The collision in which kinetic energy of the system is not conserved, is called inelastic collision.

2.4 Show that rate of change in momentum is equal to force applied. Also state Newton's second law of motion in terms of momentum.

Ans Time rate of change of momentum of a body is equal to the applied force.

Consider a body of mass 'm' moving with initial velocity v_i . Suppose an external force F acts upon it for a time t after which velocity becomes v_f . The acceleration a produced by this force is given by

$$\vec{a} = \frac{(\vec{v}_f - \vec{v}_i)}{t}$$

According to Newton's 2nd law

$$\begin{aligned}\vec{F} &= m\vec{a} \\ &= \frac{(m\vec{v}_f - m\vec{v}_i)}{t} \\ \vec{F} &= \frac{\Delta \vec{p}}{t}\end{aligned}$$

2.5 State law of conservation of linear momentum. Also state condition under which it holds.

Ans Law of conservation of linear momentum:

The total linear momentum of an isolated system remains constant.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}'_1 + m_1\vec{v}'_2$$

Condition: Law of conservation of momentum holds when the system is isolated i.e., no external agency exerts force on the system.

2.6 Show that range of a projectile is maximum at an angle of 45°.

Ans Range of projectile is given as:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

For maximum range R, the factor $\sin 2\theta = 1$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

2.7 Find the time of flight of a projectile to reach the maximum height.

Ans First equation of motion is given as

$$v_{fy} = v_{iy} + a_y t$$

At maximum height $v_{fy} = 0$, $v_{iy} = v_i \sin \theta$, $a_y = -g$

$$0 = v_i \sin \theta - gt$$

$$gt = v_i \sin \theta$$

$$t = \frac{v_i \sin \theta}{g}$$

2.8 The maximum horizontal range of a projectile is 800 m. Find the value of height attained by the projectile at $\theta = 60^\circ$.

Given Data:

$$R_{\max} = 800 \text{ m}, \theta = 60^\circ$$

To find: $h = ?$

$$\text{Solution: } R_{\max} = \frac{v_i^2}{g}$$

$$v_i^2 = R_{\max} g$$

$$v_i^2 = 800 \times 9.8 = 7840 \text{ m s}^{-1}$$

$$\text{As } h = \frac{v_i^2 \sin^2 \theta}{2g}$$

$$= \frac{7840 \times \sin^2 60^\circ}{2 \times 9.8}$$

$$= \frac{7840 \times (0.866)^2}{19.6}$$

$$h = 300 \text{ m}$$

Constructed Response Questions

2.1 Why does a hunter aiming a bird in a tree miss the target exactly at the bird?

Ans Hunter misses the target due to curved motion (projectile motion) of the bullet.

Explanation: When hunter fires aiming at a bird, bullet follows a curved path due to gravity and drops below the aim point. As a result, bullet passes beneath the bird causing the hunter to miss the target.

2.2 A person falling on a heap of sand does not hurt more as compared to a person falling on a concrete floor. Why?

Ans As, $F = \frac{\Delta p}{t}$

When person falls on sand, the time of impact increases, which reduces the effect of force that is why he does not get hurt.

When person falls on concrete, the time of impact is short, leading to greater force and injury.

2.3 State the conditions under which birds fly in air.

Ans Birds fly in air under the following conditions:

(i) The lift force must be greater than or equal to the birds' weight.

(ii) Thrust force must overcome frictional force of air.

2.4 Describe the circumstances for which velocity and acceleration of a vehicle are:

(i) **v** is zero but **a** is not zero

(ii) **a** is zero but **v** is not zero

(iii) perpendicular to one another

Ans (i) When sudden brakes are applied, then for a moment the velocity becomes zero but change in velocity is not zero. Hence **v** is zero but **a** is not zero

(ii) When a car moves with constant velocity, its acceleration will be zero. Hence **a** is zero but **v** is not zero.

(iii) When the car is moving in a circular path, the velocity is along the tangent and acceleration is directed towards the center of the circle. Therefore, velocity and acceleration are perpendicular to one another.

2.5 Describe briefly effects of air resistance on the range and maximum height of a projectile.

Ans As range and height of projectile are given by:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

In the presence of air resistance, the velocity of projectile decreases which slows down the projectile motion and hence decreases the range and height of projectile.

Comprehensive Questions

2.0 Define component of a vector. What are rectangular components? Also resolve a vector into its rectangular components.

Ans **Component of a vector:**

The component of a vector is its effective value in a given direction.

Rectangular components of a vector:

The components of a vector which are mutually perpendicular to each other are called rectangular components of vector.

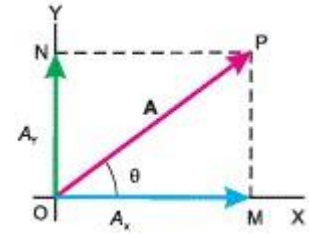
Resolution of Vector into its Rectangular Components

The process of splitting up of a vector into its components is called resolution of a vector. It is a reverse process of addition of vectors.

Explanation:

Consider a vector **A** represented by a line OP making an angle θ with x-axis. Draw perpendiculars from point P on x-axis and on y-axis such that OM and ON are the projections of OP on x-axis and y-axis respectively. OM = A_x is the rectangular component of vector **A** along x-axis and ON = A_y is the rectangular component of vector **A** along y-axis

By head to tail rule



$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y \text{ ----- (1)}$$

As, A_x and A_y are perpendicular to each other hence they are called the rectangular components of vector **A**.

Magnitude of Rectangular Component:

The magnitude of x-component of **A** is:

$$A_x = A \cos \theta \text{ ----- (2)}$$

The magnitude of y-component of **A** is:

$$A_y = A \sin \theta \text{ ----- (3)}$$

Determination of a Vector from its Rectangular Components

When the rectangular components of a vector are given, the vector can be found as follows.

Magnitude of Vector A

In $\triangle OMP$, using Pythagorean Theorem

$$(OP)^2 = (OM)^2 + (MP)^2$$

$$A = \sqrt{A_x^2 + A_y^2} \text{ ----- (4)}$$

Direction of Vector A:

The direction θ is given by

$$\tan \theta = \frac{MP}{OM}$$

$$= \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

2.1 Define and explain scalar product. Write down its important characteristics.**Ans Scalar product or dot product:**

When the product of two vectors results into a scalar quantity, it is called a scalar product. It is also called as dot product because it is represented by a dot (.) between two vectors.

Let $\vec{A}$ and $\vec{B}$ be two vectors, their scalar product can be defined as

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

where A and B are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and θ is the angle between them.

Physical Interpretation of the Dot Product:

The two vectors $\vec{A}$ and $\vec{B}$ are brought to common origin as shown in figures.

$$\vec{A} \cdot \vec{B} = A (\text{Projection of } \vec{B} \text{ on } \vec{A})$$

$$= A (\text{magnitude of component of } \vec{B} \text{ along } \vec{A})$$

$$= A(B \cos \theta)$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Similarly

$$\vec{B} \cdot \vec{A} = B (A \cos \theta)$$

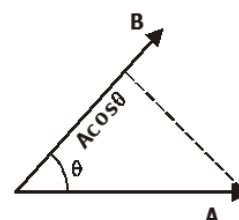
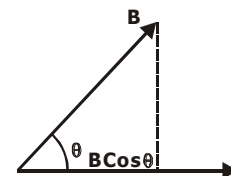
Examples:

- (i) Work is the scalar product of force $\vec{F}$ and displacement $\vec{d}$.

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

$$W = (F \cos \theta) d$$

$$W = (\text{magnitude of effective component of } \mathbf{F}) (\text{distance covered } \mathbf{d})$$



(ii) Power is the scalar product of force $\vec{F}$ and velocity $\vec{v}$.

$$P = \vec{F} \cdot \vec{v}$$

Characteristics of Scalar Product:

(1) **Scalar product is commutative.**

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

Proof:

$$\vec{A} \cdot \vec{B} = AB \cos \theta \text{ --- (1)}$$

$$\vec{B} \cdot \vec{A} = BA \cos \theta$$

$$\vec{B} \cdot \vec{A} = AB \cos \theta \text{ --- (2)} \quad (\because AB = BA)$$

Comparing equations (1) and (2), we get

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

This means the order of multiplication is irrelevant.

(2) **Scalar product of two mutually perpendicular vectors is zero.**

Let $\vec{A}$ and $\vec{B}$ be two mutually perpendicular vectors i.e. $\theta = 90^\circ$, their dot product is given by

$$\vec{A} \cdot \vec{B} = AB \cos \theta = AB \cos 90^\circ = AB (0)$$

$$\vec{A} \cdot \vec{B} = 0$$

(3) **Scalar product of two parallel or anti-parallel vectors is equal to the product of their magnitudes.**

For parallel vectors:

Let $\vec{A}$ and $\vec{B}$ be two parallel vectors i.e. $\theta = 0^\circ$, their dot product is given by

$$\vec{A} \cdot \vec{B} = AB \cos \theta = AB \cos (0) = AB (1) = AB$$

For parallel vectors, scalar product has maximum value.

For anti-parallel vectors:

Let $\vec{A}$ and $\vec{B}$ be two anti-parallel vectors i.e. $\theta = 180^\circ$, their dot product is given by

$$\vec{A} \cdot \vec{B} = AB \cos \theta = AB \cos(180) = AB (-1) = -AB$$

(4) **Self dot product:**

The self-dot product of a vector $\vec{A}$ is equal to square of its magnitude.

$$\vec{A} \cdot \vec{A} = AA \cos \theta = A^2 \cos(0)$$

$$\vec{A} \cdot \vec{A} = A^2$$

(5) **Scalar product of two vectors A and B in terms of their rectangular components**

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \text{ --- (1)}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta \text{ --- (2)}$$

Comparing equations (1) and (2), we can find the angle between two vectors.

$$AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

$$\cos \theta = \frac{A_x B_x + A_y B_y + A_z B_z}{AB}$$

$$\theta = \cos^{-1} \left(\frac{A_x B_x + A_y B_y + A_z B_z}{AB} \right)$$

2.2 Define and explain vector product of two vectors. Discuss important characteristics of vector product.

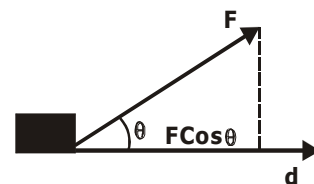
Ans Vector product or cross product:

When the product of two vectors results into a vector quantity, it is called a vector product. It is also called as cross product because it is represented by putting a ($\times$) between two vectors.

Let $\vec{A}$ and $\vec{B}$ be two vectors, their vector product is defined as:

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

Where, A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$, θ is the angle between them and $\hat{n}$ is the unit vector perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ as shown in figure below.



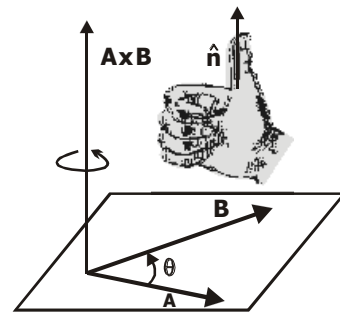
Direction of $\vec{A} \times \vec{B}$

To find the direction of $\vec{A} \times \vec{B}$ we use right hand rule.

- Join the tails of vector $\vec{A}$ and $\vec{B}$ to define their plane.
- Rotate $\vec{A}$ towards $\vec{B}$ through the smaller angle and curl your fingers in the direction of rotation. The erect thumb will give you the direction of $\vec{A} \times \vec{B}$ or $\hat{n}$.

According to right hand rule, the direction of $\vec{B} \times \vec{A}$ will be opposite to the direction of $\vec{A} \times \vec{B}$.

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



Examples:

- Torque is the vector product of position vector $\vec{r}$ and force $\vec{F}$.
$$\vec{\tau} = \vec{r} \times \vec{F}$$
- The force $\vec{F}$ on a charge q moving with velocity $\vec{v}$ in a magnetic field of strength $\vec{B}$ is given by
$$\vec{F} = q (\vec{v} \times \vec{B})$$

Characteristics of Cross Product

(1) Vector product is non commutative

Since $\vec{A} \times \vec{B}$ is not same as $\vec{B} \times \vec{A}$, the cross product is non commutative.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

(2) Cross product of two perpendicular vectors:

Let $\vec{A}$ and $\vec{B}$ be two mutually perpendicular vectors i.e. $\theta = 90^\circ$, their cross product is given as:

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = AB \sin 90^\circ \hat{n} = AB (1) \hat{n} = AB \hat{n}$$

Cross product of two perpendicular vectors has maximum magnitude.

(3) Vector product of two parallel or anti-parallel vectors:

For parallel vectors:

Let $\vec{A}$ and $\vec{B}$ be two parallel vectors i.e. $\theta = 0^\circ$, their cross product is given as

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = AB \sin 0^\circ \hat{n} = AB (0) \hat{n} = 0 \hat{n} = \vec{0}$$

For anti-parallel vectors:

Let $\vec{A}$ and $\vec{B}$ be anti-parallel vectors i.e. $\theta = 180^\circ$, their vector product is

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = AB \sin 180^\circ \hat{n} = AB (0) \hat{n} = 0 \hat{n} = \vec{0}$$

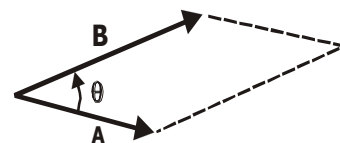
Cross product of two parallel or anti-parallel vectors is a null vector.

As a consequence, self cross product is

$$\vec{A} \times \vec{A} = \vec{0}$$

(4) Magnitude of $\vec{A} \times \vec{B}$

The magnitude of $\vec{A} \times \vec{B}$ is equal to the area of parallelogram with $\vec{A}$ and $\vec{B}$ as two adjacent sides as shown in fig.



2.3(A) Derive three equations of motion.

Ans First Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i . Suppose its velocity changes to v_f in time interval t . The acceleration produced in the body during this time interval is given as:

$$a = \frac{(v_f - v_i)}{t}$$

By rearranging, we can write

$$v_f - v_i = at$$

$$v_f = v_i + at$$

This is the first equation of motion. It correlates the final velocity attained by a body with initial velocity and time interval t , when moving with constant acceleration a .

Second Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i . Suppose its velocity changes to v_f in time interval t and covers a displacement S . Then velocity is given as rate of change of displacement:

Velocity = Displacement / Time

or Displacement = Velocity x Time

If the velocity is not constant, then we can use average velocity instead of velocity.

Thus Displacement = Average velocity x Time

$$= \frac{(\text{Initial velocity} + \text{Final velocity})}{2} \times \text{Time}$$

$$\begin{aligned} S &= \frac{(v_i + v_f)}{2} \times t \\ &= \frac{(v_i + v_i + at)}{2} \times t = \frac{(2v_i + at)}{2} \times t \\ 2S &= 2v_i t + at^2 \\ S &= v_i t + \frac{1}{2}at^2 \end{aligned}$$

This is second equation of motion.

Third Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i , which attains a final value v_f in time t . Let the displacement of the body be S during this time interval.

Displacement = $\frac{(\text{Initial velocity} + \text{Final velocity})}{2} \times \text{Time}$

$$\begin{aligned} S &= \frac{(v_i + v_f)}{2} \times t \\ 2S &= (v_i + v_f) \times t \text{ ----- (1)} \end{aligned}$$

Using first equation of motion

$$v_f = v_i + at$$

$$t = \frac{(v_f - v_i)}{a}$$

Putting the value of t in eq. (1)

$$\begin{aligned} 2S &= (v_i + v_f) \times \frac{(v_f - v_i)}{a} \\ 2aS &= (v_i + v_f) \times (v_f - v_i) \\ 2aS &= (v_f^2 - v_i^2) \end{aligned}$$

This is the third equation of motion.

2.3 Derive three equations of motion by graphical method.

Ans First Equation of Motion

First equation of motion can be derived using velocity-time graph.

Let the velocity of a body at point A is v_i which changes to v_f at point B in time interval t as shown in figure of velocity- time graph. Draw a perpendicular BD from point B on x-axis and BE from B on y-axis, such that

OA = v_i = Initial velocity of the body
 OE = BD = v_f = Final velocity of the body

From graph

$$BD = DC + CB$$

$$\text{As } DC = OA$$

$$\text{So, } BD = OA + CB$$

$$v_f = v_i + CB \text{ ----- (i)}$$

As we know that

Acceleration = Slope of velocity – time graph

$$a = \text{Slope of line AB} = \frac{CB}{AC} = \frac{CB}{t}$$

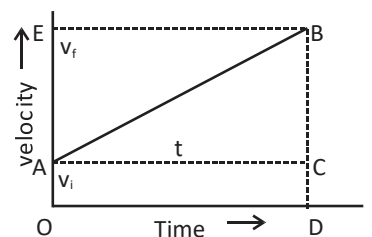
$$CB = at$$

Put the value of CB in eq.(i)

$$v_f = v_i + at$$

This is the first equation of motion.

Second Equation of Motion



Second equation of motion can be derived using velocity-time graph as follows.

Let the velocity of a body at point A is v_i which changes to v_f at point B in time interval t as shown in figure. While moving with constant acceleration a , it covers a distance S , which is equal to the area under velocity-time graph.

Distance covered = Area under velocity-time graph

S = Area of the figure OABD

S = (Area of the rectangle OACD) + (Area of triangle ABC)

$$S = (OA \times OD) + \frac{1}{2}(AC \times CB) \text{ --- (i)}$$

From graph, $OA = v_i$ and $OD = AC = t$

$$\text{So } a = \text{Slope of line AB} = \frac{CB}{AC} = \frac{CB}{t}$$

$$\text{and } CB = at$$

Putting the values of OA , OD , AC and CB in eq.(i)

$$S = (v_i \times t) + \frac{1}{2}(t \times at)$$

$$S = v_i t + \frac{1}{2}at^2$$

This is the second equation of motion.

Third Equation of motion

Let the velocity of a body at point A is v_i which changes to v_f at point B in time interval t as shown in fig of velocity-time graph. During this interval, the total distance S travelled by a body is equal to the area OABD under the graph.

Distance covered = Area under velocity-time graph

$$S = \frac{1}{2}(\text{Sum of parallel sides}) \times \text{Height}$$

$$S = \frac{1}{2}(OA + BD) \times (OD) \text{ --- (i)}$$

From graph $OA = v_i$, $BD = v_f$, $OD = t$

$$S = \frac{1}{2}(v_i + v_f) \times (t) \text{ --- (ii)}$$

From first equation of motion

$$v_f = v_i + at$$

$$at = v_f - v_i$$

$$t = \frac{v_f - v_i}{a}$$

Putting the value of t in eq.(ii)

$$S = \frac{1}{2}(v_i + v_f) \times \frac{v_f - v_i}{a}$$

$$2aS = (v_f + v_i)(v_f - v_i)$$

$$2aS = v_f^2 - v_i^2$$

This is the third equation of motion.

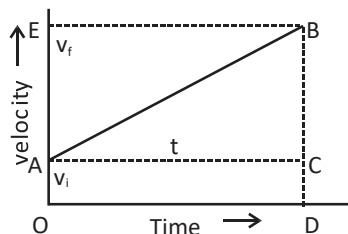
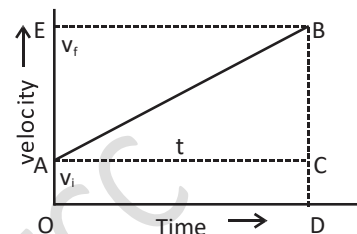
2.4 What is projectile motion? Explain.

Ans Definition: The two-dimensional motion under constant acceleration due to gravity is called projectile motion.

Examples

- i. A football kicked off by a player
- ii. A ball thrown by a cricketer
- iii. A missile fired from a launching pad

Case 1: When a body is thrown horizontally from certain height:



Consider the motion of a ball, which is thrown horizontally with velocity v_i from certain height. The body moves freely under the action of gravity as shown in figure.

Horizontal Distance:

As body is thrown horizontally, whole velocity is along x-axis i.e., $v_i = v_x$. According to Newton's first law, there will be no acceleration in horizontal direction ($a_x = 0$), since there is no horizontal force acting on it. The ball moves with constant horizontal velocity component. Hence horizontal distance x can be found by using simple relation

$$x = v_x \times t \text{ --- (1)} \quad (\because v_i = v_x)$$

Vertical Distance:

During vertical motion, the ball will accelerate under the action of force of gravity i.e., $a = g$. The initial vertical component of velocity v_{iy} is zero but it goes on increasing as the projectile moves. This component of velocity becomes maximum when projectile hits the ground.

Here, $v_{iy} = 0$, $a_y = g$, $S = y$

Using 2nd equation of motion,

$$\begin{aligned} S &= v_{iy}t + \frac{1}{2} a_y t^2 \\ y &= (0) + \frac{1}{2} a_y t^2 \\ y &= \frac{1}{2} g t^2 \text{ --- (2)} \end{aligned}$$

Case 2: When a body is thrown from the ground at a certain angle:

Suppose a projectile is fired in a direction angle ' θ ' with the x-axis with initial velocity ' v_i ' as shown in figure. The horizontal acceleration is $a_x = 0$ because no force is acting along x-axis and air resistance has been neglected. The vertical acceleration is $a_y = g$.

In this case, motion of projectile is composed of two independent motions: one along x-axis and other along y-axis. Its velocity can be resolved into two components.

Initial Horizontal component of velocity:

From Fig. $v_{ix} = v_i \cos \theta$

Initial Vertical component of velocity:

From Fig. $v_{iy} = v_i \sin \theta$

Final Horizontal component of velocity:

As there is no horizontal force (ignore air friction) acting on the projectile, therefore horizontal component of velocity remains constant throughout its motion. Using 1st equation of motion

$$\begin{aligned} v_f &= v_i + at \\ v_{fx} &= v_{ix} + a_x t \quad (\text{Along x- axis}) \\ v_{fx} &= v_i \cos \theta + (0)t \quad (\because a_x = 0) \\ v_{fx} &= v_{ix} = v_i \cos \theta \text{ --- (3)} \end{aligned}$$

Final Vertical Component of Velocity:

$$\begin{aligned} v_{fy} &= v_{iy} + a_y t \quad (\text{Along y- axis}) \\ v_{fy} &= v_{iy} + (-g) t \quad (\because a_y = g) \\ v_{fy} &= v_i \sin \theta - gt \text{ --- (4)} \end{aligned}$$

Magnitude of Final Velocity:

The magnitude of final velocity is given by

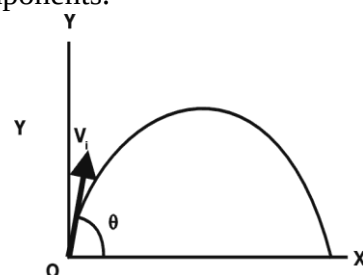
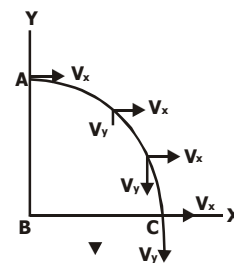
$$v = \sqrt{v_{fx}^2 + v_{fy}^2} \text{ --- (5)}$$

Direction of Velocity:

The angle θ made by the velocity with horizontal axis is given by

$$\tan \theta = \frac{v_{fy}}{v_{fx}} \text{ --- (6)}$$

2.5 Derive the following expressions for projectile motion:



- (i) **Time of flight**
- (ii) **Height attained**
- (iii) **Range for projectile**

Ans (i) Time of flight

The time taken by the body to cover the distance from point of projection to the point of landing at the same level is called time of flight.

As the projectile returns to the same level, the vertical distance covered by the projectile becomes zero.

Here, $S = h = 0$, $v_{iy} = v_i \sin\theta$, $a_y = -g$

Using second equation of motion

$$S = v_i t + \frac{1}{2} a t^2$$

$$h = v_{iy} t + \frac{1}{2} a_y t^2 \quad (\text{Along y- axis})$$

$$0 = v_{iy} t + \frac{1}{2} (-g) t^2$$

$$0 = v_i \sin\theta t - \frac{1}{2} g t^2$$

$$v_i \sin\theta = \frac{1}{2} g t$$

$$t = \frac{2v_i \sin\theta}{g}$$

(ii) Height of projectile:

The vertical distance covered by the projectile is called the height of projectile.

At maximum height, the vertical component of velocity becomes zero. For vertical motion,

$$v_{fy} = 0, S = h, a_y = -g$$

Using third equation of motion

$$2aS = v_f^2 - v_i^2$$

$$2a_y h = v_{fy}^2 - v_{iy}^2 \quad (\text{Along y- axis})$$

$$2(-g)h = 0 - (v_i \sin\theta)^2$$

$$-2gh = -v_i^2 \sin^2\theta$$

$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

(iii) Range of projectile:

The maximum horizontal distance covered by a projectile is called the range of projectile.

We know that $x = v_{ix} t$

where $x = R$, $v_{ix} = v_i \cos\theta$

So $R = v_i \cos\theta \left(\frac{2v_i \sin\theta}{g} \right)$

$$R = \frac{v_i^2 2 \sin\theta \cos\theta}{g} \quad (\text{As } 2\sin\theta \cos\theta = \sin 2\theta)$$

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Maximum Range ($R_{\max}$)

For maximum range, $\sin 2\theta = 1$, therefore

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

Hence, range is maximum at angle of 45° .

Putting value of θ ,

$$R = \frac{v_i^2 \sin 2(45)}{g}$$

$$R_{\max} = \frac{v_i^2}{g}$$

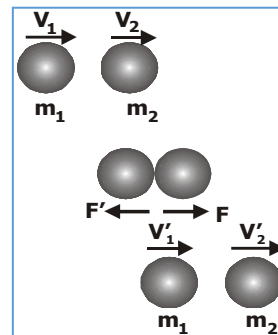
2.5 (A) State and explain law of conservation of linear momentum.

Ans Statement: The total linear momentum of an isolated system remains constant.

Mathematically,

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

Proof: Consider an isolated system of two smooth hard interacting balls of mass m_1 and m_2 . They are moving along the same straight line in the same direction with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. As $\mathbf{v}_1 > \mathbf{v}_2$, therefore both balls collide and after collision their velocities become $\mathbf{v}'_1$ and $\mathbf{v}'_2$ as shown in figure.



Change in momentum of ball of mass m_1

During the collision, ball of mass m_2 applies the impulse on ball of mass m_1 which changes its momentum. As impulse is equal to change in momentum therefore,

$$\mathbf{F}' \times t = m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1 \text{ --- (1)}$$

Change in momentum of ball of mass m_2

Similarly, ball of mass m_1 applies the impulse on m_2 which changes its momentum therefore,

$$\mathbf{F} \times t = m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2 \text{ --- (2)}$$

Adding equations (1) & (2) we get

$$\begin{aligned} (\mathbf{F}' \times t) + (\mathbf{F} \times t) &= m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1 + m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2 \\ (\mathbf{F} + \mathbf{F}')t &= m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1 + m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2 \text{ --- (3)} \end{aligned}$$

Since the action force $\mathbf{F}$ is equal and opposite to the reaction force $\mathbf{F}'$, we have $\mathbf{F}' = -\mathbf{F}$ or $\mathbf{F}' + \mathbf{F} = 0$. Hence,

$$\begin{aligned} 0 &= m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1 + m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2 \\ 0 &= (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2) \\ m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 &= m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2 \end{aligned}$$

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

2.6 Explain elastic collision in one dimension. Show that magnitude of relative velocities before and after collision are equal.

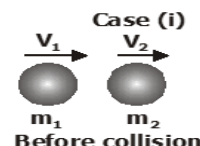
Ans Elastic Collision:

A collision in which K.E and momentum of the system is conserved is called elastic collision. e.g. collisions among the molecules of an ideal gas.

Elastic collision in one dimension:

The collision is said to be one dimensional if before and after collision the direction of motion of colliding bodies remains same.

Consider two non-rotating, smooth hard balls of masses m_1 and m_2 moving with initial velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ in the same direction along the same straight line. Let $\mathbf{v}_1 > \mathbf{v}_2$ then after some time they will collide with each other and their velocities become $\mathbf{v}'_1$ and $\mathbf{v}'_2$ along the same straight line as shown in figure.



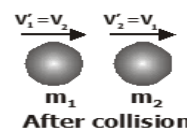
By law of conservation of momentum

Initial momentum = Final momentum

$$\begin{aligned} m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 &= m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2 \\ m_1 \mathbf{v}_1 - m_1 \mathbf{v}'_1 &= m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2 \\ m_1 (\mathbf{v}_1 - \mathbf{v}'_1) &= m_2 (\mathbf{v}'_2 - \mathbf{v}_2) \text{ --- (1)} \end{aligned}$$

By law of conservation of K. E

$$\begin{aligned} \frac{1}{2} m_1 \mathbf{v}_1^2 + \frac{1}{2} m_2 \mathbf{v}_2^2 &= \frac{1}{2} m_1 \mathbf{v}'_1^2 + \frac{1}{2} m_2 \mathbf{v}'_2^2 \\ m_1 \mathbf{v}_1^2 + m_2 \mathbf{v}_2^2 &= m_1 \mathbf{v}'_1^2 + m_2 \mathbf{v}'_2^2 \\ m_1 \mathbf{v}_1^2 + m_2 \mathbf{v}_2^2 &= m_1 \mathbf{v}'_1^2 + m_2 \mathbf{v}_2^2 \\ m_1 \mathbf{v}_1^2 - m_1 \mathbf{v}'_1^2 &= m_2 \mathbf{v}'_2^2 - m_2 \mathbf{v}_2^2 \\ m_1 (\mathbf{v}_1^2 - \mathbf{v}'_1^2) &= m_2 (\mathbf{v}'_2^2 - \mathbf{v}_2^2) \\ m_1 (\mathbf{v}_1 - \mathbf{v}'_1)(\mathbf{v}_1 + \mathbf{v}'_1) &= m_2 (\mathbf{v}'_2 - \mathbf{v}_2)(\mathbf{v}'_2 + \mathbf{v}_2) \text{ --- (2)} \end{aligned}$$



Dividing equation (2) by (1) we get:

$$\frac{m_1(v_1 - v'_1)(v_1 + v'_1)}{m_1(v_1 - v'_1)} = \frac{m_2(v'_2 - v_2)(v'_2 + v_2)}{m_2(v'_2 - v_2)}$$

$$v_1 + v'_1 = v'_2 + v_2 \text{ --- (3)}$$

$$v_1 - v_2 = v'_2 - v'_1$$

$$(v_1 - v_2) = -(v'_1 - v'_2) \text{ --- (4)}$$

where $(v_1 - v_2)$ = magnitude of relative velocity of approach of balls.

$(v'_2 - v'_1)$ = magnitude of relative velocity of separation of balls.

Conclusion:

From equation (4) we conclude that magnitude of relative velocities before and after the collision is same but in reversed direction. In other words, the magnitude of relative velocity of approach is equal to magnitude of relative velocity of separation.

2.7 Explain Elastic Collision in two dimensions.

Ans Elastic Collision in two dimensions:

Consider the motion of two balls of mass m_1 and m_2 in a straight line with velocities v_1 and v_2 respectively undergoing an elastic collision with each other as shown in fig.

Assume the bodies move off in different directions after collision with velocities $\vec{v}'_1$ and $\vec{v}'_2$ making angles θ_1 and θ_2 respectively with x-axis.

As the collision is elastic, law of conservation of momentum and law of conservation of kinetic energy will be applied. Being a vector, momentum is resolved into its rectangular components.



By conservation of Linear Momentum:

Momentum along x-direction can be written as:

Momentum before collision = momentum after collision

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1v'_1\cos\theta_1 + m_2v'_2\cos\theta_2 \text{ --- (1)}$$

Momentum along y-direction:

$$0 + 0 = m_1v'_1\sin\theta_1 - m_2v'_2\sin\theta_2$$

$$m_1v'_1\sin\theta_1 = m_2v'_2\sin\theta_2$$

By Conservation of Kinetic Energy:

Kinetic energy before collision = Kinetic energy after collision

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 \text{ --- (2)}$$

Equation (1) and (2) are useful in solving problems about elastic collisions in two dimensions.

2.8 Explain an inelastic collision in one and two dimensions.

Ans Inelastic collision: The collision in which kinetic energy of the system is not conserved, is called inelastic collision.

Inelastic collision in one Dimension

Consider two bodies having masses m_1 and m_2 , moving with velocities v_1 and v_2 along the same line such that $v_1 > v_2$. In such cases m_1 is regarded as projectile and m_2 as target. After time t both the bodies make inelastic collision and stick together, and their combined mass become $m_1 + m_2$ that moves with final velocity v_f after collision.

Conservation of Momentum:

Since the collision is perfectly inelastic, the total momentum of balls is conserved.

Total momentum of system before collision = Total momentum of system after collision

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

$$m_1 v_1 + m_2 v_2 = m_1 v_f + m_2 v_f$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_1 + m_2 v_2}{(m_1 + m_2)}$$

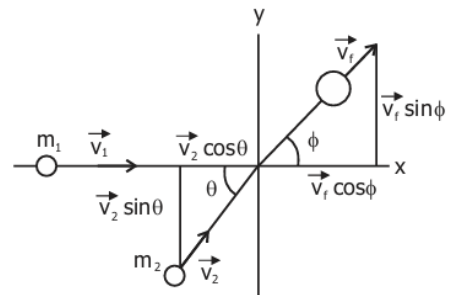
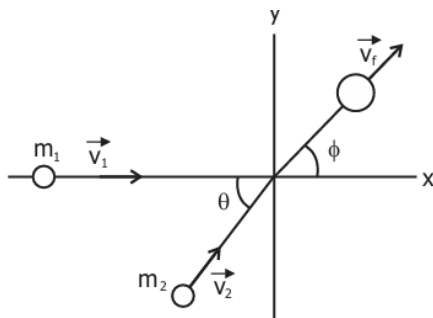
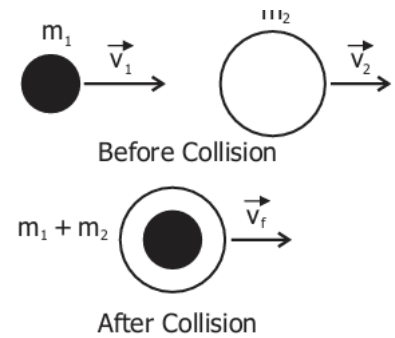
which gives the common velocity of body after inelastic collision. In a special case when the target is at rest, $v_2 = 0$, the above equation becomes:

$$v_f = \frac{m_1 v_1}{(m_1 + m_2)}$$

Inelastic Collision in Two Dimension

Let us take two rigid bodies having masses m_1 and m_2 moving with velocities, v_{1i} and v_{2i} respectively, in a two-dimensional xy-plane.

Assume that the first body is moving along x-axis while the second body moves in a direction making an angle ϕ with x-axis. Both the bodies collide at the origin as shown in figure. After collision, bodies stick together, having combined mass $M = m_1 + m_2$, which moves with velocity v_f making an angle ϕ



with x-axis.

By law of conservation of Momentum:

Momentum along x-direction can be written as:

$$m_1 v_1 \cos 0^\circ + m_2 v_2 \cos \theta = M v_f \cos \phi$$

$$m_1 v_1 + m_2 v_2 \cos \theta = M v_f \cos \phi$$

Momentum along y-direction can be written as:

$$m_1 v_1 \sin 0^\circ + m_2 v_2 \sin \theta = M v_f \sin \phi$$

$$m_2 v_2 \sin \theta = M v_f \sin \phi$$

By Law of conservation of Kinetic Energy:

Initial kinetic energy of system is given by formula:

$$(K.E)_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Final kinetic energy when both objects stuck together will be:

$$(K.E)_f = \frac{1}{2} M v_f^2 \quad (\because M = m_1 + m_2)$$

Loss in Kinetic Energy

Since the collision is inelastic, the loss in K.E is represented by

$$\Delta K.E = (K.E)_i - (K.E)_f$$

Numerical Problems

2.1 The magnitude of cross and scalar products of two vectors are $4\sqrt{3}$ and 4 respectively. Find the angle between the vectors.

Given data:

$$|\vec{A} \times \vec{B}| = 4\sqrt{3}$$

$$|\vec{A} \cdot \vec{B}| = 4$$

To find: $\theta = ?$

Solution: We know that

$$|\vec{A} \times \vec{B}| = AB \sin \theta \text{ -----(1)}$$

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta \text{ -----(2)}$$

Dividing equation (1) by (2),

$$\frac{|\vec{A} \times \vec{B}|}{|\vec{A} \cdot \vec{B}|} = \frac{AB \sin \theta}{AB \cos \theta}$$

Putting values,

$$\frac{\sin \theta}{\cos \theta} = \frac{4\sqrt{3}}{4}$$

$$\tan \theta = \frac{4\sqrt{3}}{4}$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = 60^\circ$$

Ans

2.2 A helicopter is ascending vertically at the rate of 19.6 m s^{-1} . When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

Given data:

$$v_i = 19.6 \text{ m s}^{-1}$$

$$v_f = 0$$

$$h_1 = 156.8 \text{ m}$$

$$g = 9.8 \text{ m s}^{-2}$$

To find:

$$t = ?$$

Solution: For upward motion of stone

$$v_f = v_i + gt$$

$$0 = 19.6 + (-9.8) t_1$$

$$0 = 19.6 - 9.8 t_1$$

$$9.8 t_1 = 19.6$$

$$t_1 = \frac{19.6}{9.8}$$

$$t_1 = 2 \text{ s}$$

$$h_2 = v_i t_1 + \frac{1}{2} g t_1^2$$

$$= (19.6 \times 2) + \frac{1}{2} (-9.8)(2)^2$$

$$= 39.2 - 19.6$$

$$h_2 = 19.6 \text{ m}$$

For downward motion of stone

$$h = h_1 + h_2$$

$$h = 156.8 + 19.6$$

$$h = 176.4 \text{ m}$$

$$h = v_i t_2 + \frac{1}{2} g t_2^2$$

$$176.4 = 0 + \frac{1}{2} (9.8) t_2^2 \quad (\because v_i = 0)$$

$$176.4 = 4.9 t_2^2$$

$$t_2^2 = \frac{176.4}{4.9}$$

$$t_2^2 = 36$$

$$t_2 = 6 \text{ s}$$

Thus,

$$t = t_1 + t_2$$

$$t = 2 + 6$$

$$t = 8 \text{ s}$$

Ans.

2.3 If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then prove that $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Given data:

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

To find: Prove that vector $\vec{A}$ and $\vec{B}$ are perpendicular $\vec{A} \cdot \vec{B} = 0$

Solution:

It is given that

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

Squaring on both sides,

$$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

$$\begin{aligned}
(A+B) \cdot (A+B) &= (A-B) \cdot (A-B) \\
A \cdot A + A \cdot B + B \cdot A + B \cdot B &= A \cdot A - A \cdot B - B \cdot A + B \cdot B \\
A \cdot A + 2A \cdot B + B \cdot B &= A \cdot A - 2A \cdot B + B \cdot B & (\because A \cdot B = B \cdot A) \\
A^2 + B^2 + 2A \cdot B - A^2 - B^2 + 2A \cdot B - 2A \cdot B &= 0 & (\because A \cdot A = A^2, B \cdot B = B^2) \\
4A \cdot B &= 0 \\
A \cdot B &= 0 \\
AB \cos \theta &= 0 \\
\theta &= \cos^{-1}(0) \\
\theta &= 90^\circ
\end{aligned}$$

Hence, the angle between A and B is 90° .

2.4 A body of mass M at rest explodes into 3 pieces, two of which of mass $\frac{M}{4}$ each are thrown off in perpendicular directions with velocities of 3 m s^{-1} and 4 m s^{-1} respectively. Find the velocity of 3rd piece with which it will be flown away.

Given data:

Total mass of the body before explosion = M

Initial velocity = $v_i = 0 \text{ m s}^{-1}$

Mass of 1st and 2nd piece after explosion = $m_1 = m_2 = \frac{M}{4}$

Velocity of 1st piece = $v_1 = 3 \text{ m s}^{-1}$ (along x – axis)

Velocity of 2nd piece = $v_2 = 4 \text{ m s}^{-1}$ (along y – axis)

Mass of 3rd piece = $m_3 = M - \frac{M}{4} - \frac{M}{4} = \frac{M}{2}$

To find:

Velocity of 3rd piece = $v_3 = ?$

Solution:

Momentum of 1st piece = $p_1 = m_1 v_1 = \frac{M}{4} \times 3 = \frac{3M}{4}$ (x – direction)

Momentum of 2nd piece = $p_2 = m_2 v_2 = \frac{M}{4} \times 4 = M$ (y – direction)

Momentum of 3rd piece = $p_3 = m_3 v_3 = \frac{M}{2} \times v_3$

The resultant momentum of the first two pieces is given by

$$\begin{aligned}
p_R &= \sqrt{p_1^2 + p_2^2} \\
p_R &= \sqrt{\left(\frac{3M}{4}\right)^2 + (M)^2} \\
p_R &= \sqrt{\frac{9M^2}{16} + (M)^2} \\
p_R &= \sqrt{\frac{9M^2}{16} + \left(\frac{16M}{16}\right)^2} \\
p_R &= \sqrt{\frac{25M^2}{16}} \\
p_R &= \frac{5M}{4}
\end{aligned}$$

The momentum of 3rd piece must equal to the negative of the resultant momentum,

$$p_3 = -p_R = -\frac{5M}{4}$$

Putting value of p_3 , we get

$$\begin{aligned}
\frac{M}{2} \times v_3 &= -\frac{5M}{4} \\
v_3 &= -\frac{5M}{4} \times \frac{2}{M} = -\frac{5}{2} \\
v_3 &= -2.5 \text{ m s}^{-1} ; \text{ opposite to the resultant velocity vector of two pieces.}
\end{aligned}$$

2.5 A cricket ball is hit upward with velocity of 20 m s^{-1} at an angle of 45° with the ground. Find its: (a) time of flight (b) maximum height (c) how far away it hits the ground

Given data:

$$v_i = 20 \text{ m s}^{-1}$$

$$\theta = 45^\circ$$

$$g = 9.8 \text{ m s}^{-2}$$

To find:

(a) $t = ?$

(b) $h = ?$

(c) $R = ?$

Solution

(a) $t = \frac{2 v_i \sin \theta}{g} = \frac{2 \times (20) \times \sin 45}{9.8} = 2.9 \text{ s}$

(b) $h = \frac{v_i^2 \sin^2 \theta}{2g} = \frac{(20)^2 \times \sin^2 45}{2 \times 9.8} = 10.2 \text{ m}$

(c) $R = \frac{v_i^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 2(45)}{9.8} = 41 \text{ m}$

2.6 A 20 g ball hits the wall of a squash court with a constant force of 50 N. If the time of impact of force is 0.50 s, find the impulse.

Given data:

$$m = 20 \text{ g} = 0.020 \text{ kg}$$

$$F = 50 \text{ N}$$

$$\Delta t = 0.50 \text{ s}$$

To find:

$$I = ?$$

Solution:

$$I = F \times \Delta t$$

$$I = 50 \times 0.50$$

$$I = 25 \text{ N s} \quad \text{Ans}$$

2.7 A ball is kicked by a footballer. The average force on the ball is 240 N, and the impact lasts for a time interval of 0.25 s.

(a) Calculate change in momentum

(b) State the direction of change in momentum

Given data:

$$F_{\text{ave}} = 240 \text{ N}$$

$$\Delta t = 0.25 \text{ s}$$

To find:

(a) $\Delta p = ?$

(b) direction of change in momentum (Δp) = ?

Solution:

(a) $F = \frac{\Delta p}{\Delta t}$
 $\Delta p = F \times \Delta t$
 $= 240 \times 0.25$
 $\Delta p = 60 \text{ N s} \quad \text{Ans}$

(b) The direction of change in momentum is same as the direction of applied force.

2.8 An aeroplane is moving horizontally at a speed of 200 m s^{-1} at a height of 8 km to drop a bomb on a target. Find horizontal distance from the target at which the bomb should be released.

Given data:

$$v_{ix} = 200 \text{ ms}^{-1}$$

$$v_{iy} = 0$$

$$y = h = 8.0 \text{ km} = 8000 \text{ m}$$

$$g = 9.8 \text{ ms}^{-2}$$

To find:

$$x = ?$$

Solution:

$$y = \frac{1}{2} g t^2$$
$$\sqrt{\frac{2y}{g}} = t$$

$$t = \sqrt{\frac{2 \times 8000}{9.8}} = 40.40 \text{ s}$$

$$x = v_{ix} t = 200 \times 40.40$$

$$x = 8080 \text{ m} = 8.08 \text{ km} \quad \text{Ans}$$

2.9 Why does range R of a projectile remain the same when angle of projection is changed from θ to $\theta' = 90^\circ - \theta$. Also show that for complementary angles of projection the ratio $\frac{R}{R'}$ is equal to 1.

Given data: $\theta' = 90^\circ - \theta$

To Prove: $\frac{R}{R'} = 1$

Solution:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

For complementary angles

$$R' = \frac{v_i^2 \sin 2\theta'}{g} = \frac{v_i^2 \sin 2(90^\circ - \theta)}{g}$$

$$R' = \frac{v_i^2 \sin(180^\circ - 2\theta)}{g} = \frac{v_i^2 \sin 2\theta}{g} \quad (\because \sin(180^\circ - 2\theta) = \sin 2\theta)$$

$$\frac{R}{R'} = \frac{\frac{v_i^2 \sin 2\theta}{g}}{\frac{v_i^2 \sin 2\theta}{g}} = 1$$

2.10 A trolley of mass 1.0 kg is moving with velocity 1.0 m s⁻¹ collides with a similar trolley at rest:

- (i) after collision, the 1st trolley comes to rest whereas the second starts moving with velocity of 1.0 m s⁻¹ in the same direction. Show that it is an example of an elastic collision.
- (ii) after the collision, they stick together and move away with a velocity of 0.5 m s⁻¹. Show that it is an example of an inelastic collision.

Given data:

$$m_1 = m_2 = 1.0 \text{ kg}$$

$$v_1 = 1.0 \text{ m s}^{-1}$$

$$v_2 = 0$$

$$v'_1 = 0$$

$$v'_2 = 1.0 \text{ m s}^{-1}$$

$$v = 0.5 \text{ m s}^{-1}$$

To find:

Solution:

- (i) By law of conservation of K.E

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_1v'^2_2$$

$$\frac{1}{2}(1 \times 1^2) + \frac{1}{2}(1 \times 0) = \frac{1}{2}(1 \times 0) + \frac{1}{2}(1 \times 1^2)$$

$$0.5 \text{ J} = 0.5 \text{ J}$$

Energy is also conserved, so this is elastic collision.

- (ii) By law of conservation of energy

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2$$

$$\frac{1}{2}(1 \times 1^2) + \frac{1}{2}(1 \times 0) = \frac{1}{2}(1 + 1) \times 0.5$$

$$0.5 \text{ J} \neq 0.25 \text{ J}$$

As law of conservation of energy is not valid, so it is an inelastic collision

2.11 A railway wagon of mass 4×10^4 kg moving with velocity of 3 m s^{-1} collides with another wagon of mass 2×10^4 kg which is at rest. They stick together and move off together. Find their combined velocity.

Given data:

$$m_1 = 4 \times 10^4 \text{ kg}$$

$$v_1 = 3 \text{ ms}^{-1}$$

$$m_2 = 2 \times 10^4 \text{ kg}$$

$$v_2 = 0$$

To find:

Combined velocity = $v = ?$

Solution: By law of conservation of momentum

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$(4 \times 10^4) \times 3 + (2 \times 10^4) \times 0 = (4 \times 10^4 + 2 \times 10^4) v$$

$$12 \times 10^4 = (6 \times 10^4) v$$

$$v = 2 \text{ ms}^{-1} \quad \text{Ans}$$

2.12 A car with mass 575 kg moving at 15.0 m s^{-1} smashes into the rear end of a car with mass 1575 kg moving at 5 m s^{-1} in the same direction.

(a) What is the final velocity if the wrecked car locks together?

(b) How much kinetic energy is lost in the collision?

Given data:

$$m_1 = 575 \text{ kg}$$

$$v_1 = 15.0 \text{ m s}^{-1}$$

$$m_2 = 1575 \text{ kg}$$

$$v_2 = 5.0 \text{ m s}^{-1}$$

To find:

(a) combined velocity = $v = ?$

(b) K.E lost = $\Delta K.E = ?$

Solution:

(a)

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$(575 \times 15) + (1575 \times 5) = (575 + 1575) v$$

$$16500 = 2150 v$$

$$v = 7.67 \text{ m s}^{-1} \quad \text{Ans}$$

(b)

$$\Delta K.E = K.E_i - K.E_f$$

$$\Delta K.E = \left[\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right] - \left[\frac{1}{2} (m_1 + m_2) v^2 \right]$$

$$\Delta K.E = \left[\left(\frac{1}{2} 575 \times 15^2 \right) + \left(\frac{1}{2} 1575 \times 5^2 \right) \right] - \left[\frac{1}{2} (575 + 1575) (7.67)^2 \right]$$

$$\Delta K.E = 84375 \text{ J} - 63231.5 \text{ J}$$

$$\Delta K.E = 2.11 \times 10^4 \text{ J} \quad \text{Ans}$$

Chapter 3

Exercise

Circular and Rotational Motion

MCQs

- 3.1 The ratio of angular speed of minute's hand and hour's hand of a watch is:
(A) 1:6 (B) 6:1 (C) 1:12 (D) 12:1
- 3.2 A body travelling in a circle at constant speed:
(A) has constant velocity (B) has an inward radial acceleration
(C) is not accelerated (D) has an outward radial acceleration
- 3.3 A stone at the end of long string is whirled in vertical circle at a constant speed. The tension in the string will be minimum when the stone is:
(A) at the top of the circle (B) half way down
(C) at the bottom of circle (D) anywhere in the circle
- 3.4 Every point of rotating rigid body has:
(A) same angular velocity (B) same linear velocity
(C) same linear acceleration (D) same linear distance
- 3.5 The minimum velocity necessary to put a satellite into the orbit is called:
(A) terminal velocity (B) critical velocity (C) artificial velocity (D) angular velocity
- 3.6 An astronaut is orbiting around the Earth in a large capsule. Then,
(A) he will be in a state of weightlessness with respect to capsule
(B) he is freely falling towards the Earth
(C) a ball projected at an angle has a straight-line path as observed by him
(D) all the above
- 3.7 An object in uniform circular motion makes 10 revolutions in 2 seconds. Which of the following statement is true?
(A) its period is 2.0 s (B) its period is 20 s
(C) its frequency is 5 Hz (D) its frequency is 0.2 Hz
- 3.8 A man inside the artificial satellite feels weightlessness because the force of attraction due to the earth is:
(A) zero at pole (B) balanced by the force of attraction due to the moon
(C) equal to the centripetal force (D) non-effective due to some particular design of satellite
- 3.9 A bottle of soda water is grasped from the neck and swung briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?
(A) Near the bottom
(B) In the middle of bottle
(C) Bubbles remain distributed throughout the volume of the bottle
(D) Near the neck of the bottle
- 3.10 The moment of inertia of a body depends upon:
(A) mass of the body and its distribution about axis of rotation
(B) volume of the body (C) kinetic energy of the body
(D) angular momentum of the body

Answer Key					
Q.No.	Option	Q.No.	Option	Q.No.	Option
3.1	D	3.5	B	3.9	D
3.2	B	3.6	D	3.10	A
3.3	A	3.7	C		
3.4	A	3.8	C		

Short Answer Questions

- 3.1 State second law of motion in case of rotation.

Ans The rate of change of angular momentum is equal to applied torque

$$\tau = I \alpha$$

$$T = \frac{\Delta L}{\Delta t}$$

3.2 What is the effect of changing the position of a diver while diving in the pool?

Ans The diver changes his body position to conserve angular momentum according to the following relation.

$$I_1 \omega_1 = I_2 \omega_2$$

For stretched position of the diver, moment of inertia I_1 is greater and angular velocity ω_1 will be smaller. When the diver is in tuck position then his moment of inertia I_2 decreases and angular velocity ω_2 will increase. Due to this, the product of I and ω in both situations remains constant.

3.3 How do we get butter from the milk by using centrifuge?

Ans A cream separator works on the principle of centrifuge. Since milk is a mixture of light and heavy particles, when it is rotated the light particles gather near the axis of rotation while heavy particles go outward and hence butter can easily be separated from milk. In this machine, milk is whirled rapidly.

3.4 Mass is a measure of inertia in linear motion. What is its analogue in rotational motion? Describe briefly.

Ans The quantity $I = mr^2$ is known as moment of inertia and represented by I . The moment of inertia plays the same role in angular motion as the mass in linear motion. Inertia depends on mass only but moment of inertia depends on mass and square of moment arm.

3.5 Why is it harder for a car to take turn at higher speed than at lower speed?

Ans It is harder for a car to take turn at higher speed than at lower speed.

While turning, a car require centripetal force, given by

$$F_c = \frac{mv^2}{r}$$

This force is provided by the friction between road and tyres. At higher speed the car needs more centripetal force to take turn so if the required force is not available, car may skid. Hence turning is harder at higher speed.

3.6 What are the benefits of using rear wheels of heavy vehicles consisted of double tyres?

Ans The rear wheels consisting of double tyres:

- i) Improve the load bearing capacity.
- ii) Improve stability by increasing surface area.
- iii) Distribute weight more evenly.

3.7 When a moving car turns around a corner to the left, in what direction do the occupants tend to fall? Explain briefly.

Ans Occupants will tend to fall towards right (Opposite direction).

According to law of inertia, body wants to keep its motion in a straight line. When car turns left, it changes the direction, but body tries to keep going straight which is towards the right side relative to the car.

3.8 Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?

Ans According to Newton's 2nd Law, when a force acts on a body it produces acceleration in the same direction. Since centripetal force acts towards the center. So, the acceleration also acts towards the center.

3.9 How does an astronaut feel weightlessness while orbiting from the Earth in a spaceship?

Ans As the spaceship falls freely, the normal reaction force acting on astronaut inside spaceship is zero because the centripetal force becomes equal to the weight of the body. Thus astronaut experiences weightlessness.

Constructed Response Questions

3.1 If angular velocity of different particles of a rigid body is constant, will the linear velocity of these particles be also constant?

Ans No, the linear velocity of these particles will not be constant.

As different particles have different distances from the axis of rotation so linear velocity will not be same by magnitude as well as direction. As direction is continuously changing so linear velocity will not be same.

3.2 A loaf of bread is lying on rotating plate. A crow takes away the loaf of bread and the rotation of the plate increase. Why?

Ans The rotation of plate increases to conserve the angular momentum.

$$I_1 \omega_1 = I_2 \omega_2$$

As crow takes away the bread, the mass decreases which decreases the moment of inertia. Hence, the angular velocity will increase.

3.3 Why do we tumble when we take the sharp turn with large speed?

Ans We tumble when we take a sharp turn due to inertia and centripetal force. A large centripetal force is required to change direction but body continues linear motion due to inertia. As a result, we lose balance and tumble outward.

3.4 What will be time period of a simple pendulum in an artificial satellite at a certain height?

Ans In an artificial satellite gravitational acceleration will be zero due to weightlessness.

As
$$T = 2\pi \sqrt{\frac{l}{g}}$$

Hence time period of simple pendulum is infinite.

3.5 Is the motion of a satellite in its orbit, uniform or accelerated?

Ans The motion of satellite in its orbit is accelerated. A change in velocity means there is centripetal acceleration which is directed towards the center of the Earth. So, even if the speed is uniform, the satellite is still undergoing continuous acceleration due to the change in direction of velocity.

3.6 What are the advantages that radian has been preferred as SI unit to degree?

Ans Radian is defined as ratio of arc length and radius. This makes it pure number without any dimension, and it makes the calculation easy in trigonometry.

$$\theta = \frac{s}{r}$$

3.7 In uniform circular motion, what are the average velocity and average acceleration for one revolution? Explain.

Ans
$$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$$

For one revolution, change in angular displacement is zero i.e. $\Delta\theta = 0$

Hence
$$\omega_{ave} = \frac{0}{\Delta t} = 0$$

Similarly,
$$\alpha_{ave} = \frac{\Delta\omega}{\Delta t}$$

For one revolution, change in angular velocity is zero i.e. $\Delta\omega = 0$

Hence,
$$\alpha_{ave} = \frac{0}{\Delta t} = 0$$

3.8 In a rainstorm with a strong wind, what determines the best position to hold an umbrella?

Ans Angle the umbrella to block the rain while tilting it into the wind and holding it lower to minimize wind resistance. Adjust constantly as rain and wind directions change.



3.9 A ball is just supported by a string without breaking. If it is whirled in a vertical circle, it breaks. Explain Why.

Ans When the ball is hanging, the tension T in the string is equal to the weight of the ball (mg) i.e.,
$$T = mg$$

However, when the ball is whirled in a vertical circle, especially at lowest point, the string must provide both the weight of the ball and centripetal force to keep it in circular motion. The tension T becomes

$$T = mg + \frac{mv^2}{r}$$

Thus the string breaks due to the additional centripetal force requirement, which increases the net tension beyond its limit.

3.10 How is the centripetal force supplied in the following cases:

- A satellite orbiting around the Earth?**
- A car taking a turn on a level road?**

- Ans a) Gravity provides centripetal force.
b) Friction provides the centripetal force.

Comprehensive Questions

3.1 What is meant by angular momentum? Explain the law of conservation of angular momentum with daily life examples.

Ans **Angular Momentum:**

A particle is said to possess angular momentum about a reference axis, if it so moves that its angular position changes relative to that reference axis.

OR

The cross product of position vector $\vec{r}$ and linear momentum $\vec{p}$ is called angular momentum.

Mathematically:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = r p \sin\theta \hat{n}$$

S.I Unit:

$$\text{kg m}^2 \text{s}^{-1} \text{ or J s}$$

Law of conservation of Angular Momentum:

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{\text{total}} = L_1 + L_2 + \dots = \text{constant}$$

Examples of Conservation of Angular Momentum:

i) **Man diving from a diving board**

A diver jumping from a springboard has to take a few somersaults in air before touching the water surface. After leaving the springboard, he curls his body by rolling arms and legs in. Due to this, his moment of inertia decreases, and he spins in midair with a large angular velocity. When he is about to touch the water surface, he stretches out his arms and legs. He enters the water at a gentle speed and gets a smooth dive. This is an example of the law of conservation of angular momentum.

ii) **The spinning Ice skater**

An ice skater can increase his angular velocity by folding arms and bringing the stretched leg close to the other leg. By doing so, he decreases his moment of inertia. As a result, angular speed increases. When he stretches his hands and a leg outward, the moment of inertia increases and hence angular velocity decreases.

iii) **A person holding some weight in his hands standing on a turntable**

A person is standing on a turntable with heavy mass (dumb-bell) in his hands stretched out on both sides. As he draws his hands inward, his angular speed at once increases. This is because the moment of inertia decreases on drawing the hands inwards, which increases the angular speed.

iv) **Flywheel**

A flywheel is a mechanical device that consists of a heavy wheel with an axle. It is used to store rotational energy, smooth out output fluctuations, and provides stability in a wide range of applications, such as bicycles and other vehicles, industrial machinery, gyroscopes, ships, and space crafts. When a flywheel spins, its angular momentum resists changes to its orientations, maintaining stability. This is useful in systems that need precise control over their orientation without external interference.

v) A **gyroscope** is a device which is used to maintain its orientation relative to the Earth's axis or resists changes in its orientation. It consists of a mounted flywheel pivoted in supporting rings as shown in Fig. 3.23. It works on the basis of law of conservation of angular momentum due to its large moment of inertia. When the gyroscope spins at a large angular speed, it gains large angular momentum. It is then difficult to change the orientation of the gyroscope's rotational axis due to its large moment of inertia. A change in orientation requires a change in its angular momentum. To change the direction of a large angular momentum, a corresponding large torque is

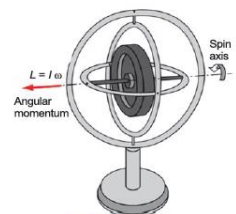
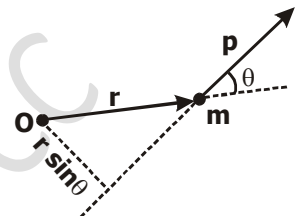


Fig. 3.23: The gyroscope



Fig. 3.24

required. Even if gyroscope is tilted (Fig. 3.24), it still keeps levitated without falling. Hence, it is a reason why a gyroscope can be used to maintain orientation.

3.2 Show that orbital angular momentum is; $L = I\omega$.

Ans Consider a symmetric rigid body rotating about a fixed axis as shown in fig. It is made up of n small masses $m_1, m_2, \dots, m_n$ at distances $r_1, r_2, \dots, r_n$ from the axis of rotation. Each mass has the same angular velocity ω as that of rigid body. The angular momentum of all the small masses $m_1, m_2, \dots, m_n$ is given as:

$$\begin{aligned} L_1 &= m_1 r_1^2 \omega \\ L_2 &= m_2 r_2^2 \omega \\ &\vdots \\ L_n &= m_n r_n^2 \omega \\ L &= L_1 + L_2 + \dots + L_n \\ L &= (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2) \omega \\ L &= \left(\sum_{i=1}^n m_i r_i^2 \right) \omega \quad \text{where } I = \sum_{i=1}^n m_i r_i^2 \\ L &= I \omega \end{aligned}$$

Where I is the moment of inertia of the rigid body about the axis of rotation.

3.3 Define moment of inertia. Prove that torque acting on rotating rigid body is equal to the product of its moment of inertia and angular acceleration.

Ans Moment of Inertia: The ability of body to resist the change in its state of rotational motion or rest about an axis.

Mathematically,

$$I = mr^2$$

Units: The SI Unit of moment of inertia is kg m^2 .

Torque acting on rotating rigid body

Consider a rigid body made up of large n small pieces of masses $m_1, m_2, m_3, \dots, m_n$ at distances $r_1, r_2, r_3, \dots, r_n$ respectively from the axis of rotation O . Since the body is rigid, all the masses are rotating with the same angular acceleration α . Then, the magnitude of torque acting on m_1 is:

$$\tau_1 = m_1 r_1^2 \alpha$$

Similarly, on mass m_2 :

$$\tau_2 = m_2 r_2^2 \alpha$$

$\vdots$

And on mass m_n :

$$\tau_n = m_n r_n^2 \alpha$$

$$\tau = \tau_1 + \tau_2 + \tau_3 + \dots + \tau_n$$

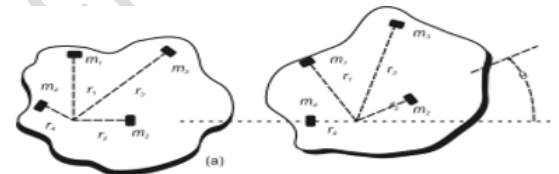
$$\tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \dots + m_n r_n^2 \alpha$$

$$\tau = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \alpha$$

$$\tau = \left(\sum_{i=1}^n m_i r_i^2 \right) \alpha$$

$$\tau = I \alpha$$

where $I = \sum_{i=1}^n m_i r_i^2$ is called moment of inertia.



3.4 What are artificial satellites? Calculate the minimum time period necessary to put a satellite into the orbit near the surface of the Earth.

Ans Artificial Satellites: Man-made objects which revolve around the earth in a particular orbit are called artificial satellites.

Explanation: Consider a low flying satellite which is put into the orbit by rockets. As satellite is moving in the circular orbit, it has centripetal acceleration a_c which is given by

$$a_c = \frac{v^2}{r} \text{ --- (1)}$$

Orbital velocity: The velocity with which satellite moves in a particular orbit around the earth is called orbital velocity. As satellite is low flying and the centripetal acceleration is provided by the gravity, hence in this case; $r = R$ (h is ignored as it is very low) and $a_c = g$. Putting these values in equation (1), we get

$$g = \frac{v^2}{R}$$

$$v^2 = gR$$

$$v = \sqrt{gR} \text{ --- (2)}$$

As, $R = 6.4 \times 10^6 \text{ m}$ and $g = 9.8 \text{ m s}^{-2}$

Putting these values in equation (2), we get

$$v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.9 \times 10^3 \text{ m s}^{-1}$$

$$v = 7.9 \text{ km s}^{-1}$$

This is the minimum velocity necessary to put a satellite into the orbit and is called critical velocity.

Time Period: Time required by satellite to complete one revolution around the Earth is called time period. When satellite completes one revolution, it covers a distance equal to circumference of orbit, i.e., $S = 2\pi R$ and $t = T$. Therefore,

$$S = vt \quad \text{or} \quad t = \frac{S}{v}$$

$$T = \frac{2\pi R}{v}$$

$$= \frac{2 \times 3.14 \times 6.4 \times 10^6}{7.9 \times 10^3} = 5060 \text{ s}$$

$$T = \frac{5060}{60} \text{ min} = 84 \text{ min. approx.}$$

The acceleration is inversely proportional to square of distance from the centre of Earth. Therefore, when height h of the satellite above the surface of Earth increases then $r = R + h$ also increases. For higher satellites, the speed decreases and time period increases. Height h for close orbiting satellites is 400 km and twenty-four such satellites form GPS (Global Positioning System).

3.5 Define orbital velocity and derive an expression for the same.

Ans. Orbital Velocity: The velocity of an object revolving around the Earth in a particular orbit is called orbital velocity.

$$v = \sqrt{\frac{GM}{r}}$$

Explanation: Consider a satellite of mass m_s revolving around the Earth with velocity v in an orbit of radius r . The centripetal force acting on the satellite is provided by the gravitational force of earth.

$$\text{As, } F_g = \frac{GMm_s}{r^2} \quad \text{and} \quad F_c = \frac{m_s v^2}{r}$$

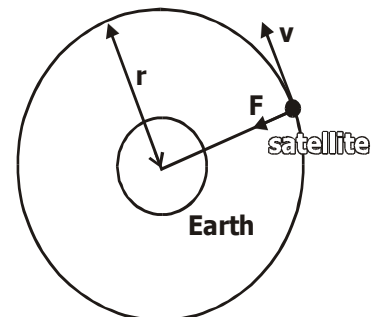
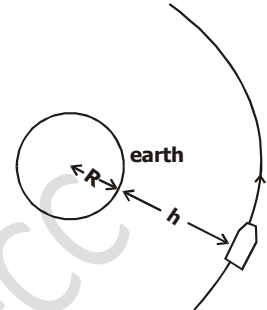
$$\text{So, } F_c = F_g$$

$$\frac{m_s v^2}{r} = \frac{GMm_s}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

where G is gravitational constant, and M is mass of Earth.



The above equation shows that mass of satellite does not have any effect on the satellite's orbital velocity.

Significance: Any satellite orbiting at a distance r from the earth's centre must have orbital speed given by the equation

$$v = \sqrt{\frac{GM}{r}}.$$

Any speed less than this will bring the satellite tumbling back to the Earth.

3.6 Write a note on artificial gravity. Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.

Ans Artificial Gravity: The gravity provided to the spacecraft by spinning it about its own axis is called artificial gravity.

Explanation: Consider a spacecraft having radius ' R ' falling freely in space. Under this condition all the objects and astronauts inside it become weightless. It is difficult for astronauts to stay long and perform experiments in this state of weightlessness. Therefore, an artificial gravity must be provided. For this purpose, the spacecraft is set in rotation about its own axis with angular speed ' ω '. The astronauts are then pressed towards the outer rim. They exert the force on the floor of spacecraft in much the same way as on the earth. The reactional force of the floor balances them in the spacecraft. The centripetal acceleration of the spacecraft is given by

$$a_c = \frac{v^2}{R}$$

Since $v = R\omega$ therefore, above equation becomes

$$a_c = \frac{R^2\omega^2}{R}$$

$$a_c = R\omega^2 \text{ ----- (1)}$$

As,

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T}$$

Also

$$\frac{1}{T} = f$$

$$\omega = 2\pi f$$

Putting value of ω in equation (1), we have

$$a_c = R(2\pi f)^2$$

$$a_c = R4\pi^2 f^2$$

$$f^2 = \frac{a_c}{4\pi^2 R}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{a_c}{R}}$$

The force of gravity provides the required centripetal acceleration, therefore $a_c = g$,

$$\text{So } f = \frac{1}{2\pi} \sqrt{\frac{g}{R}}$$

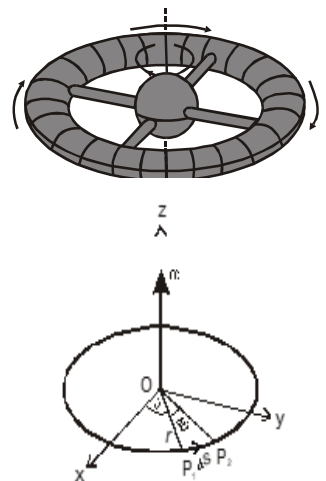
When spaceship rotates with this frequency about its own axis then artificial gravity like Earth is provided to the astronauts.

3.7 Prove that: (i) $v = r\omega$ (ii) $a = r\alpha$

Ans: (i) $v = r\omega$

Proof: Consider a point P moving along a circle of radius r with a linear velocity v , whereas the line OP rotates with angular velocity. Suppose, in a time interval Δt , the point P moves through a distance, $P_1P_2 = \Delta S$, along the boundary of the circle, and the line OP has angular displacement $\Delta\theta$ during this interval. The relation between ΔS and $\Delta\theta$ is given as:

$$\Delta S = r\Delta\theta \text{(1)}$$



Dividing both sides of Eq. (1) by Δt

$$\frac{\Delta S}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

Applying limit $\Delta t \rightarrow 0$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$$

Thus,

$$v = r \omega$$

(ii) $a = r \alpha$

Proof: The relation between angular and linear velocity is given as:

$$v = r \omega$$

$$\Delta v = r \Delta \omega$$

Dividing both sides by Δt

$$\frac{\Delta v}{\Delta t} = r \frac{\Delta \omega}{\Delta t}$$

Applying limit $\Delta t \rightarrow 0$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$$

Thus,

$$a = r \alpha$$

Numerical Problems

3.1 A laser beam is directed from the Earth to the moon. The beam spreads over a diameter of 2.50 m at the moon surface, what is divergence angle of the beam? The distance of moon from the Earth is 3.8×10^8 m.

Given data:

$$S = 2.50 \text{ m}$$

$$r = 3.8 \times 10^8 \text{ m}$$

To find:

$$\theta = ?$$

Solution:

According to formula:

$$S = r\theta$$

$$\theta = \frac{S}{r}$$

$$= \frac{2.50}{(3.8 \times 10^8)}$$

$$\theta = 6.6 \times 10^{-9} \text{ rad}$$

Ans

3.2 A car is moving with a speed of 108 km h^{-1} . If its wheel has a diameter of 60 cm, find its angular speed in rad s^{-1} and rev s^{-1} .

Given data:

$$v = 108 \text{ kmh}^{-1} = \frac{108 \times 1000}{3600} = 30 \text{ ms}^{-1}$$

$$d = 60 \text{ cm} = 0.60 \text{ m}$$

$$r = \frac{d}{2} = \frac{0.60}{2} = 0.30 \text{ m}$$

To find:

$$\omega = ?$$

Solution:

$$v = r\omega$$

$$\omega = \frac{v}{r} = \frac{30}{0.30} = 100 \text{ rad s}^{-1}$$

For revolution per second

$$(\because 1 \text{ rad} = \frac{1}{2\pi} \text{ rev})$$

$$\omega = \frac{100}{2\pi} = 15.9 = 16 \text{ rev s}^{-1}$$

Ans

3.3 An electric motor is running at $1800 \text{ rev min}^{-1}$. On switching off, it comes to rest in 20 s. If angular retardation is uniform, find the number of revolutions it makes before stopping.

Given data:

$$\omega_i = 1800 \text{ rev min}^{-1} = \frac{1800}{60} = 30 \text{ rev s}^{-1}$$

$$\omega_f = 0$$

$$t = 20 \text{ sec}$$

To find:

$$\theta = ?$$

Solution:

$$\theta = \omega_{\text{ave}} \times t$$

$$\theta = \left(\frac{\omega_i + \omega_f}{2} \right) \times t$$

$$\theta = \frac{30+0}{2} \times 20$$

$$\theta = 300 \text{ rev}$$

Ans

3.4 A string 0.5 m long holding a stone can withstand maximum tension of 35.6 N. Find the maximum speed at which a stone of 0.5 kg mass can be whirled with it in a vertical circle.

Given data:

$$r = 0.5 \text{ m}$$

$$T = 35.6 \text{ N}$$

$$m = 0.5 \text{ kg}$$

To find:

$$v = ?$$

Solution:

$$T - w = F_c$$

$$T - mg = \frac{mv^2}{r}$$

$$\frac{(T - mg) \times r}{m} = v^2$$

$$v^2 = \frac{(35.6 - 0.5 \times 9.8)}{0.5} \times 0.5$$

$$v^2 = 30.7 \text{ ms}^{-1}$$

$$v = 5.5 \text{ ms}^{-1}$$

Ans

3.5 The flywheel of an engine is rotating at 2100 rev min⁻¹ when the power source is shut off. What torque is required to stop it in 3 minutes? The moment of inertia of the flywheel is 36 kg m².

Given data:

$$\omega_i = 2100 \text{ rev min}^{-1}$$

$$\omega_i = \frac{2100 \times 2\pi}{60} \text{ rads}^{-1}$$

$$\omega_i = 220 \text{ rads}^{-1}$$

$$t = 3 \text{ min} = 3 \times 60 = 180 \text{ s}$$

$$\omega_f = 0$$

$$I = 36 \text{ kgm}^{-2}$$

To find:

$$\tau = ?$$

Solution:

$$\tau = I\alpha$$

$$\tau = I \left(\frac{\omega_f - \omega_i}{t} \right)$$

$$= 36 \left(\frac{0 - 220}{180} \right)$$

$$\tau = -44 \text{ N m}$$

Negative sign shows the retardation so,

$$\tau = 44 \text{ N m}$$

Ans

3.6 What is the moment of inertia of a 200 kg sphere whose diameter is 60 cm?

Given data:

$$m = 200 \text{ kg}$$

$$d = 60 \text{ cm}$$

$$d = 0.60 \text{ m}$$

$$r = \frac{d}{2} = \frac{0.60}{2} = 0.30 \text{ m}$$

To find:

$$I = ?$$

Solution: For sphere of radius r and mass m ,

$$I = \frac{2}{5}mr^2$$

$$I = \frac{2}{5}[200 \times (0.30)^2]$$

$$I = 7.2 \text{ kg m}^2$$

Ans

3.7 A satellite is orbiting the Earth at an altitude of 200 km. Assuming the Earth's radius is 6400 km, calculate the orbital speed of the satellite.

Given data:

$$h = 200 \text{ km}$$

$$R = 6400 \text{ Km}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$$

$$M = 6 \times 10^{24} \text{ kg}$$

To find:

$$v = ?$$

Solution:

$$v = \sqrt{\frac{GM}{r}}$$

$$r = R + h$$

$$r = 6400 + 200 = 6600 \text{ km} = 6600 \times 10^3 \text{ m}$$

$$r = 6.6 \times 10^6 \text{ m}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.6 \times 10^6}}$$

$$v = 7.77 \times 10^3 \text{ m s}^{-1}$$

$$v = 7.77 \text{ km s}^{-1}$$

Ans

3.8 A space station has a radius of 20 m and rotates at an angular velocity of 0.5 rad s^{-1} . What is the artificial gravity experienced by the astronauts on the space station?

Given data:

$$R = 20 \text{ m}$$

$$\omega = 0.5 \text{ rad s}^{-1}$$

To Find:

$$a_c = ?$$

Solution:

$$a_c = R\omega^2$$

$$a_c = 20 \times (0.5)^2$$

$$a_c = 5 \text{ m s}^{-2}$$

Ans

3.9 A bicycle wheel has an angular momentum of $10 \text{ kg m}^2 \text{ s}^{-1}$ and angular velocity of 2 rad s^{-1} . Find the value of its moment of inertia.

Given data:

$$L = 10 \text{ kg m}^2\text{s}^{-1}$$

$$\omega = 2 \text{ rad s}^{-1}$$

To Find:

$$I = ?$$

Solution:

$$L = I\omega$$

$$I = \frac{L}{\omega}$$

$$I = \frac{10}{2}$$

$$I = 5 \text{ kg m}^2$$

Ans

3.10 A diver comes off a board with arms straight up and legs straight down, giving him a moment of inertia of 18 kg m^2 about his rotation axis. Then tucks into a small ball, decreasing his moment of inertia to 3.6 kg m^2 . While tucked, he makes two complete rotations in 1.0 second. If he had not tucked at all, how many revolutions would he have made in 1.5 s from board to water?
Given data:

$$\begin{aligned} I_1 &= 18 \text{ kg m}^2 \\ I_2 &= 3.6 \text{ kg m}^2 \\ \theta_2 &= 2 \text{ rev} \\ t_2 &= 1 \text{ s} \\ t_1 &= 1.5 \text{ s} \end{aligned}$$

To Find:

$$\theta_1 = ?$$

Solution:

$$\begin{aligned} I_1 \omega_1 &= I_2 \omega_2 \\ I_1 \frac{\theta_1}{t_1} &= I_2 \frac{\theta_2}{t_2} \\ \theta_1 &= I_2 \frac{\theta_2}{t_2} \times \frac{t_1}{I_1} \\ \theta_1 &= \left(\frac{3.6 \times 2}{1} \right) \times \frac{1.5}{18} \\ &= 7.2 \times 0.083 \\ &= 0.59 \text{ rev} \\ \theta_1 &= 0.6 \text{ rev} \end{aligned}$$

Ans

Chapter 4

Exercise

Work and Energy

MCQs

- 4.1 A 1 kg mass has potential energy of 1 joule relative to the ground when it is at a height of:
(A) 0.102 m (B) 1 m (C) 9.8 m (D) 32 m
- 4.2 An iron sphere whose mass is 30 kg has the same diameter as an aluminum sphere whose mass is 10.5 kg. The spheres are simultaneously dropped from a cliff. When they are 10 m away from the ground, they have identical:
(A) accelerations (B) momentums (C) potential energies (D) kinetic energies
- 4.3 A body at rest may have:
(A) speed (B) velocity (C) momentum (D) energy
- 4.4 The height above the ground of a child on a swing varies from 0.5 m of his lowest point to 1.5 m at the highest point. The maximum speed of the child is approximately:
(A) 1.5 m s^{-1} (B) 4.4 m s^{-1} (C) 9.8 m s^{-1} (D) depends upon child's mass
- 4.5 When a ball is thrown vertically upward and then falls back to the ground, which force can be considered conservative in this scenario?
(A) Air resistance (B) Gravity
(C) Friction between ball and air (D) Contact force with hand
- 4.6 According to work-energy principle in linear motion, the work done on body is equal to:
(A) change in K.E (B) change in P.E (C) Zero (D) sum of K.E and P.E
- 4.7 Power of lamp is 6 W. How much energy does a lamp give out in 2 min?
(A) 12 J (B) 20 J (C) 3 J (D) 720 J
- 4.8 A dry battery can deliver 3000 J energy to a 2 W small electric motor before the battery is exhausted. For how many minutes does the battery run?
(A) 1500 min (B) 100 min (C) 50 min (D) 25 min
- 4.9 The kinetic energy acquired by a mass m after travelling a fixed distance from rest under the action of a constant force is directly proportional to:
(A) $\sqrt{m}$ (B) $1/\sqrt{m}$ (C) m (D) independent of mass
- 4.10 A body moves a distance of 10 m along a straight line under the action of 5 N force. If the work done is 25 J, the angle which the force makes with the direction of motion of the body?
(A) 0° (B) 30° (C) 60° (D) 90°

Answer Key

Q.No.	Option	Q.No.	Option	Q.No.	Option
4.1	A	4.5	B	4.9	C
4.2	A	4.6	A	4.10	C
4.3	D	4.7	D		
4.4	B	4.8	D		

Short Answer Questions

- 4.1 Why is electrical power required at all when the elevator is descending? Why should be there a limit on the number of passengers in this case?

Ans When the elevator is descending, the electrical power is provided at a constant speed to control it, stop it from free-falling and move it upward if needed.
There should be a limit on number of passengers in the lift to balance the net force and to keep it within limits of braking system.

- 4.2 A body is being raised to a height H from surface of the Earth. What is the sign of work done by both (body and the Earth)? Justify.

Ans The work done is given by,

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd \cos \theta$$

Work done by the body:

Here

$$\theta = 0^\circ$$

$$W = mgH \cos 0^\circ = mgH (1) (\because \cos 0^\circ = 1)$$

$$W = mgH$$

Hence work done by the body is positive.

Work done by the Earth:

Here

$$\theta = 180^\circ$$

$$W = mgH \cos 180^\circ = mgH(-1) \quad (\because \cos 180^\circ = -1)$$

$$W = -mgH$$

Hence work done by the Earth is negative.

4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during fall? Justify.**Ans** Yes, the total mechanical energy is conserved when a body falls towards the Earth in air.

While falling, P.E of the body is converted into K.E & part of the P.E is used in doing work against air resistance i.e.,

Loss in P. E = gain in K.E + work done against air resistance

$$mgh = \frac{1}{2} m v^2 + fh$$

4.4 Calculate power of a crane in kilowatt which lifts a mass of 1000 kg to a height of 100 m in 20 seconds.**Ans Given:**

$$m = 1000 \text{ kg}, h = 100 \text{ m}, t = 20 \text{ sec}$$

To find:

$$P = ?$$

Solution:

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{1000 \times 9.8 \times 100}{20} = 49000 \text{ W} = 49 \text{ kW}$$

4.5 A trolley of mass 1500 kg carrying sandbags of 500 kg is moving uniformly with a speed of 40 km h⁻¹ on a frictionless track. After some time, sand starts leaking out of whole sand bags on the road at a rate of 0.05 kg s⁻¹. What is the speed of the trolley after entire sand bags are empty?**Ans Given:**

$$m_i = 1500 + 500 = 2000 \text{ kg}$$

$$v_i = \frac{40 \times 1000}{3600} = 11.11 \text{ m s}^{-1}$$

$$m_f = 1500 \text{ kg}$$

To find: $v_f = ?$

$$\text{Solution: } p_i = m_i v_i = 2000 \times 11.11 = 22,200 \text{ N s}$$

$$p_f = m_f v_f = 1500 \times v_f$$

By law of conservation of Momentum,

$$p_i = p_f$$

$$22,200 = 1500 \times v_f$$

$$v_f = \frac{22,200}{1500} = 14.81 \text{ m s}^{-1}$$

4.6 Give absolute and gravitational units of work in M.K.S and C.G.S systems.**Ans Absolute Units:**

$$\text{MKS: joule (J)} = \text{N m} = \text{kg m}^2 \text{ s}^{-2}$$

$$\text{CGS = erg (erg)} = \text{dyne cm} = \text{g cm}^2 \text{ s}^{-2}$$

Gravitational Units:

$$\text{MKS: kilogram-meter (kg m)}$$

$$\text{CGS: gram-centimeter (g cm)}$$

4.7 A body dropped from a height of H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate work done by air friction.

$$\text{Given: Height} = H, \quad v_f = v = 1.2\sqrt{gH}$$

$$\text{Solution: } \Delta \text{K.E} = \frac{1}{2} m v^2 - 0 = \frac{1}{2} m v^2$$

Total work done is,

$$\begin{aligned}
 \text{Loss of P.E} &= \text{Gain in K.E.} + \text{Work done against friction} \\
 mgH &= \Delta K.E + W_f \\
 W_f &= mgH - \frac{1}{2} m v^2 \\
 &= mgH - \frac{1}{2} m (1.2 \sqrt{gH})^2 \\
 &= mgH - \frac{1}{2} m (1.44gH) = mgH - 0.72mgH \\
 W_f &= 0.28 mgH
 \end{aligned}$$

4.8 A bicycle has a K.E of 150 J. What K.E would the bicycle have if it had:

i) same mass but has speed double?

ii) three times mass and was moving with one half of the speed?

Ans i) As $K.E' = \frac{1}{2} m v'^2$
 New speed $v' = 2v$
 $K.E = \frac{1}{2} m (2v)^2 = 4 \times \left(\frac{1}{2} m v^2 \right) = 4 \times 150 = 600 \text{ J} \quad (\because K.E. = 150 \text{ J})$

ii) Here, $m' = 3m, v' = \frac{v}{2}$
 $K.E' = \frac{1}{2} m' v'^2 = \frac{1}{2} (3m) \left(\frac{v}{2} \right)^2 = \frac{3}{4} \times \frac{1}{2} m v^2$
 $= \frac{3}{4} \times 150$
 $K.E' = 112.5 \text{ J}$

4.9 What will be the effect on K.E of the body having mass m, moving with velocity v when its momentum becomes double? Justify.

Ans As, $K.E = \frac{1}{2} m v^2$

Also, $K.E = \frac{p^2}{2m}$

Here, $p' = 2p$

$K.E' = \frac{(2p)^2}{2m} = 4 \left(\frac{p^2}{2m} \right) = 4 K.E$

4.10 Does the international space-station have gravitational P.E or kinetic energy or both? Explain.

Ans The international space station (ISS) has both gravitational P.E and K.E.

The ISS is in Earth's gravitational field, so it has gravitational P.E. It stays in orbit due to the balance between its K.E & gravitational P.E. Both forms of energy are crucial in orbital motion of the space station.

Constructed Response Questions

4.1 When will you say that a force is conservative? Give two conditions.

Ans If the work done by a force in moving an object between two points is independent of path followed or the work done in a closed path is zero, the force is called conservative force.

Conditions:

1. Total Energy (K.E+ P.E) is conserved
2. It depends upon the initial & final position.

4.2 A light and a heavy body have same linear momentum, which one has greater K.E?

Ans $K.E. = \frac{1}{2} m v^2$
 $= \frac{1}{2m} m^2 v^2 = \frac{1}{2m} (mv)^2 \quad (\because p = mv)$
 $K.E. = \frac{p^2}{2m}$
 $K.E \propto \frac{1}{m}$

Hence, the light body will have greater kinetic energy.

4.3 A motorcycle is running with constant speed on a horizontal track. Is any work being done on the motorcycle, if no net force acting on it?

Ans The work done is,

$$W = \mathbf{F} \cdot \mathbf{d}$$

$$W = Fd \cos \theta$$

As there is no net force acting on the motorcycle i.e. $F = 0$

$$W = (0)d \cos \theta$$

$$W = 0$$

Hence, the work done on the motorcycle is zero.

4.4 A force acts on a ball moving with 14 m s^{-1} speed and brings its speed to 6 m s^{-1} . Has the force done positive or negative work? Explain your answer.

Ans As we know the work-energy principle is:

Work done = Change in kinetic energy

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$= \frac{1}{2}m(6^2 - 14^2)$$

$$= \frac{1}{2}m(36 - 196)$$

$$= \frac{1}{2}m(-160)$$

$$= -\frac{160}{2}m$$

$$W = -80m$$

Hence the work done by the force is negative.

The speed of the ball decreases, the change in K.E is negative. Hence work done is negative.

4.5 A slow moving truck can have more kinetic energy than a fast-moving car. How is this possible?

Ans Since, the mass of the truck is much greater than mass of the car. Therefore, difference of masses is much greater than the difference in velocities of truck and car. Hence K.E of the truck is greater than the car.

$$\text{K.E} = \frac{1}{2}m v^2$$

4.6 Why work done against friction is non-conservative in nature? Explain briefly.

Ans Work done against friction is non-conservative because it depends on the path followed. The work done against friction is dissipated as heat and this heat can't be fully recovered. Therefore, total mechanical energy is not conserved.

4.7 Does wind contain K.E? Explain.

Ans Yes wind contain K.E.

It can be calculated from formula $\text{K.E} = \frac{1}{2}mv^2$ where 'm' is the mass of air and 'v' is the velocity of air. Wind turbines use this energy to generate electricity.

Comprehensive Questions

4.1 Define K.E. and derive an expression for the same.

Ans Kinetic Energy: It is the energy possessed by the body due its motion.

$$\text{K.E} = \frac{1}{2}mv^2$$

Proof: Consider a car running with a constant speed on a road. If its engine is switched OFF, it will still cover some distance before stopping. As long as it is moving, it is doing work against the force of friction of the road. In other words, during this interval, it will exert a force equal in magnitude to the force of friction f . Let the distance travelled before coming to rest be d , then the work done by the car would be fd . This work is done by the car due to its motion and is equal to its kinetic energy.

From second law of motion

$$F=ma$$

As the car slows down and finally stops, its acceleration a is negative because it is produced by force of friction f acting opposite to the direction of motion. Thus,

$$f = -ma \dots\dots\dots (1)$$

Using third equation of motion,

$$2aS = v_f^2 - v_i^2$$

$$S = \frac{v_f^2 - v_i^2}{2a} \dots\dots\dots (2)$$

Here, $v_i = v$, $v_f = 0$, $S = d$
Putting in eq.(2) we get

$$d = \frac{-v^2}{2a} \dots\dots\dots (3)$$

Multiplying equation (1) and (3) we get,

$$f d = -ma \times \frac{-v^2}{2a}$$

$$f d = \frac{1}{2} mv^2$$

$$\text{Kinetic energy} = \frac{1}{2} mv^2$$

S.I units of kinetic energy is joule (J).

4.2 How is work done by a: (i) constant force (ii) variable force

Ans (i) Work done by a constant force:

The scalar product of force and displacement is called work.

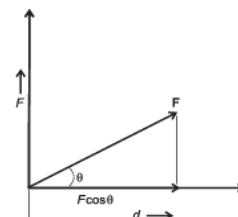
$$W = \vec{F} \cdot \vec{d} = F d \cos\theta$$

Consider a constant force $\vec{F}$ pulling a body of mass m . It makes an angle θ with the displacement $\vec{d}$ produced in the body along x-axis as shown in figure.

$$W = \vec{F} \cdot \vec{d}$$

$$= F d \cos\theta$$

$$= (F \cos\theta) (d)$$



$$W = (\text{magnitude of component of force } \vec{F} \text{ in the direction of } \vec{d}) \times (\text{distance moved})$$

Therefore, work can also be defined as the product of magnitude of component of force in the direction of $\vec{d}$ and distance moved.

Properties of Work:

- (i) Work is a scalar quantity.
- (ii) If $\theta < 90^\circ$, the work is done and it is positive.
- (iii) If $\theta = 90^\circ$, no work done.
- (iv) If $\theta > 90^\circ$, the work is negative.
- (v) SI unit of work is N m, also known as joule (J)

1 joule:

1 joule = (1 newton) (1 meter)

Work is one joule when a force of one newton displaces a body through a distance of one meter along the direction of force. (Erg is smaller unit of work 1 joule = 10^7 erg).

(vi) Dimension of work = $[ML^2 T^{-2}]$

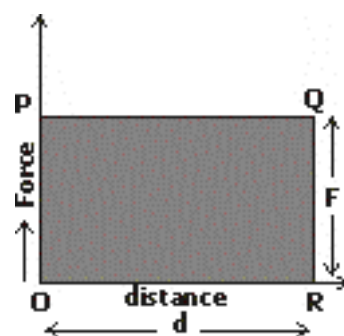
Work from Force-Displacement Graph:

Consider a constant force 'F' acting on a body through a distance 'd'. Graph is plotted between 'F' and 'd'. This is called force- displacement graph. In this case, F is taken along y-axis, d is taken along x- axis. When force is constant the Fd graph is a horizontal straight line as shown in the figure.

When force is acting along displacement then θ is equal to zero. Therefore,

$$W = F d \cos\theta$$

$$= F d \cos 0^\circ$$



$$W = F d \text{ -----(1)}$$

Also

$$\begin{aligned} \text{Area under } Fd\text{-graph} &= (\text{Length})(\text{width}) \\ &= F d \text{ -----(2)} \end{aligned}$$

From equation (1) & (2)

$$W = \text{Area}$$

Work done = Area under the force-displacement graph

(ii) Work done by a variable force:

Consider a variable force moves a particle from point 'P' to point 'Q' along a curved path, as shown in the figure.

To find work divide the path into 'n' small intervals of displacements $\Delta \vec{d}_1, \Delta \vec{d}_2, \Delta \vec{d}_3, \dots, \Delta \vec{d}_n$ the forces during these intervals are $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$. These intervals are so small that force remains constant during each interval.

$$W_1 = \vec{F}_1 \cdot \Delta \vec{d}_1 = F_1 \Delta d_1 \cos \theta_1$$

$$W_1 = (F_1 \cos \theta_1) \Delta d_1$$

$$W_2 = (F_2 \cos \theta_2) \Delta d_2$$

$$\text{-----}$$

$$W_n = (F_n \cos \theta_n) \Delta d_n$$

$$W_{\text{total}} = W_1 + W_2 + \dots + W_n$$

$$W_{\text{total}} = (F_1 \cos \theta_1) \Delta d_1 + (F_2 \cos \theta_2) \Delta d_2 + \dots +$$

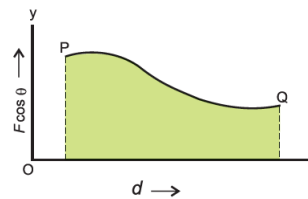
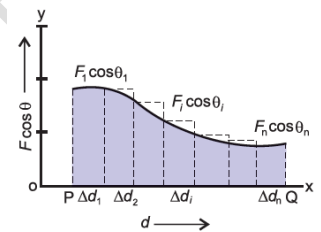
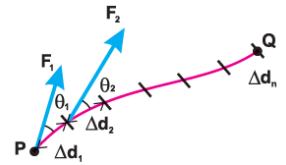
$$(F_n \cos \theta_n) \Delta d_n$$

$$W_{\text{total}} = \sum_{i=1}^n F_i \cos \theta_i \Delta d_i$$

Graphical Method:

When a variable force acts on the body to move it from P point to Q point, the graph is plotted between ' $F \cos \theta$ ' and ' d '. $F \cos \theta$ is taken along y-axis and d is taken along x-axis. The total displacement is divided into n equal and very small displacements such that each displacement $\Delta d \rightarrow 0$. The total work done is equal to the area under force-displacement graph. Therefore, the sum of areas of all the rectangles is equal to the total work done on the body by the variable force.

$$W_{\text{total}} = \lim_{\Delta d \rightarrow 0} \sum_{i=1}^n F_i \cos \theta_i \Delta d_i$$



4.3 Define conservative field. Show that gravitational field is conservative in nature.

Ans Conservative Field:

A field in which the work done is independent of the path followed by an object moving between two points is called conservative field.

Or

The field in which the work done in a closed path is zero is called conservative field.

Examples:

- Gravitational Field
- Electric Field
- Magnetic Field

Explanation:

Consider a body of mass m being displaced with constant velocity from A to B along three different paths in the gravitational field. As the body is displaced in a gravitational field so work is done by the gravitational force. In this case the gravitational force is equal to weight mg of the body. i.e. $F = w = mg$

Work done along Path 1 (ADB):

The work done by gravitational force along path ADB can be divided into two parts i.e. path AD and path DB

$$W_{ADB} = W_{AD} + W_{DB} \text{ --- (A)}$$

$$W_{AD} = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

Along this path gravitational force and displacement are perpendicular to each other i.e. $\theta = 90^\circ$

$$W_{AD} = Fd \cos 90^\circ$$

$$W_{AD} = Fd (0) \quad (\because \cos 90^\circ = 0)$$

$$W_{AD} = 0$$

Along this path gravitational force and displacement are antiparallel to each other i.e. $\theta = 180^\circ$

$$W_{DB} = Fd \cos 180^\circ \quad (\because F = w = mg)$$

$$W_{DB} = mgh (-1) \quad (\because \cos 180^\circ = -1)$$

$$W_{DB} = -mgh$$

From equation (A)

$$W_{ADB} = (0) + (-mgh)$$

$$W_{ADB} = -mgh \text{ --- (1)}$$

Work done along Path 2 (ACB):

The work done by gravitational force along path ACB can be divided into two parts i.e. path AC and path CB

$$W_{ACB} = W_{AC} + W_{CB} \text{ --- (B)}$$

Along this path gravitational force and displacement are perpendicular to each other i.e. $\theta = 90^\circ$

$$W_{CB} = Fd\cos\theta$$

$$W_{CB} = Fd \cos 90^\circ$$

$$W_{CB} = Fd (0) \quad (\because \cos 90^\circ = 0)$$

$$W_{CB} = 0$$

Along this path gravitational force and displacement are antiparallel to each other i.e. $\theta = 180^\circ$

$$W_{AC} = Fd\cos\theta$$

$$W_{AC} = Fd \cos(180^\circ) \quad (\because F = mg)$$

$$W_{AC} = mgh (-1) \quad (\because \cos 180^\circ = -1)$$

$$W_{AC} = -mgh$$

From equation (B)

$$W_{ACB} = (0) + (-mgh)$$

$$W_{ACB} = -mgh \text{ --- (2)}$$

Work done along Path 3 (AB):

The curved path AB is broken into a series of horizontal and vertical steps as shown in figure.

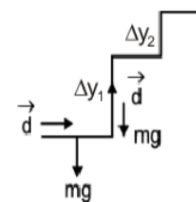
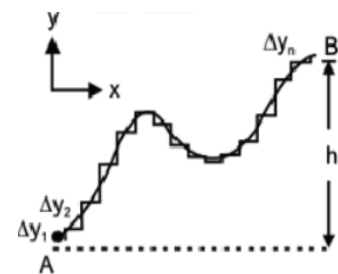
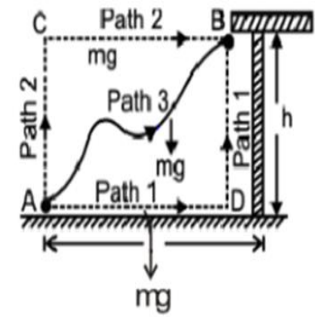
Work done along horizontal steps:

Work done along horizontal steps is zero because gravitational force and horizontal displacements are perpendicular to each other i.e. $\theta = 90^\circ$.

Work done along vertical steps:

Let vertical distance from A to B is divided in n small vertical steps $\Delta y_1, \Delta y_2, \dots, \Delta y_n$. The total work done from A to B is equal to sum of work done during these small steps.

$$W_{AB} = (-mg\Delta y_1) + (-mg\Delta y_2) + \dots + (-mg\Delta y_n)$$



$$W_{AB} = -mg (\Delta y_1 + \Delta y_2 + \dots + \Delta y_n)$$

$$\text{Here } h = \Delta y_1 + \Delta y_2 + \dots + \Delta y_n$$

$$W_{AB} = -mgh \quad (3)$$

From equations (1), (2) & (3) it is clear that work done along different paths in a gravitational field is the same. Hence work done is independent of path followed. Therefore, gravitational field is a conservative field.

Work done in a gravitational field around a closed path ACBA

The work done along a closed path ACBA is given by:

$$W_{ACBA} = W_{AC} + W_{CB} + W_{BA}$$

From the previous calculation we have $W_{AC} = -mgh$, $W_{CB} = 0$, $W_{BA} = mgh$,

$$W_{ACBA} = -mgh + 0 + mgh$$

$$W_{ACBA} = 0$$

Hence, the total work done in the gravitational field along the closed path is zero. It also proves that gravitational field is a conservative field.

4.4 What is meant by absolute P.E? Derive an expression for absolute P.E.

Ans Definition:

The amount of work done by the gravitational force on an object in displacing it from that position to infinity where the force of gravity becomes zero is called absolute potential energy.

$$U = -\frac{GMm}{R}$$

Explanation:

Consider a body of mass 'm' lifted from point 1 to N in the gravitational field. The distance between points 1 and N is very very large. Therefore, gravitational force does not remain constant because $F \propto \frac{1}{r^2}$. Due to this reason, we cannot calculate the work done by gravitational force by applying formula of potential energy mgh. We divide the distance between points 1 and N into small steps each of length Δr so that force remains constant for each small step, as shown in figure. The total work done can be calculated by adding the work done during all these steps.

Derivation of Expression:

Let r_1 and r_2 be the distances of points 1 and 2 respectively, from centre O of the Earth. If r is the distance between the centre of the first step and centre of the Earth, then it is given by

$$r = \frac{r_1 + r_2}{2} \quad (1)$$

$$\text{If } \Delta r = r_2 - r_1 \quad (2)$$

$$\text{then } r_2 = \Delta r + r_1$$

Putting value of r_2 in equation (1) we get

$$r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$r = \frac{2r_1 + \Delta r}{2} = \frac{2r_1}{2} + \frac{\Delta r}{2}$$

$$r = r_1 + \frac{\Delta r}{2} \quad \text{By squaring both sides, we get}$$

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2 = r_1^2 + \frac{\Delta r^2}{4} + 2r_1 \frac{\Delta r}{2}$$

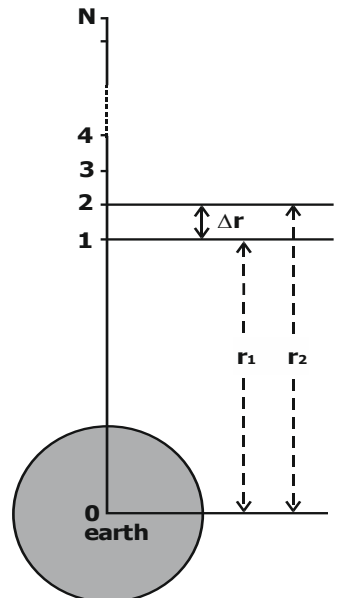
$$r^2 = r_1^2 + r_1 \Delta r \quad \left(\frac{\Delta r^2}{4} \text{ is neglected because } \Delta r^2 \ll r_1^2\right)$$

Substituting the value of Δr from equation (2)

$$r^2 = r_1^2 + r_1 (r_2 - r_1)$$

$$r^2 = r_1^2 + r_1 r_2 - r_1^2$$

$$r^2 = r_1 r_2 \quad (3)$$



The gravitational force F at the centre of this step is

$$F = G \frac{Mm}{r^2} \dots\dots\dots (4)$$

Where m = mass of object, M = mass of the Earth, G = Gravitational constant

Putting the value of r^2 from equation (3) in equation (4),

$$F = G \frac{Mm}{r_1 r_2} \dots\dots\dots (5)$$

As this force is assumed to be constant during the interval Δr , so the work done from point 1 to 2

$$W_{1 \rightarrow 2} = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos 180^\circ$$

$$W_{1 \rightarrow 2} = F \Delta r (-1) = -F \Delta r$$

Putting the values of Δr and F from equations (2) and (5) in above equation,

$$W_{1 \rightarrow 2} = -GMm \frac{(r_2 - r_1)}{r_1 r_2}$$

$$W_{1 \rightarrow 2} = -GMm \left(\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2} \right)$$

$$W_{1 \rightarrow 2} = -GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Similarly, the work done during other steps is given by

$$W_{2 \rightarrow 3} = -GMm \left(\frac{1}{r_2} - \frac{1}{r_3} \right)$$

$$\dots\dots\dots$$

$$\dots\dots\dots$$

$$W_{N-1 \rightarrow N} = -GMm \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right)$$

Hence, the total work done in moving a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{\text{total}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + \dots\dots\dots + W_{N-1 \rightarrow N}$$

$$W_{\text{total}} = -GMm \left[\left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \left(\frac{1}{r_2} - \frac{1}{r_3} \right) \dots\dots\dots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N} \right) \right]$$

$$W_{\text{total}} = -GMm \left(\frac{1}{r_1} - \frac{1}{r_N} \right)$$

If the point N is at infinite distance from the Earth, So, $r_N = \infty$ and $\frac{1}{r_N} = \frac{1}{\infty} = 0$

$$W_{\text{Total}} = -\frac{GMm}{r_1}$$

The general expression for absolute gravitational P.E. of a body at a distance r from centre of the Earth is $U = -\frac{GMm}{r}$

If the body is displaced from surface of the Earth, then $r = R$ (radius of the Earth)

$$\text{Absolute potential energy} = U = -\frac{GMm}{R}$$

Significance:

The negative sign shows that earth gravitational field for mass m is attractive when r increases, U becomes less negative i.e. U increases. It means when we raise a body above the surface of earth its P.E. increases.

4.5 State and explain work-energy theorem in a resistive medium.

Statement: The change in kinetic energy of an object is equal to the work done on it by a net force.

$$W = \text{Change in K.E} = (K.E)_f - (K.E)_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

This is also known as work-energy principle.

Proof:

Whenever work is done on a body, it increases its energy. For example, if a force F acts on the body of mass m , initially moving with velocity v_i , through a distance d and increases its velocity to v_f then the acceleration produced will be

$$\begin{aligned} 2ad &= v_f^2 - v_i^2 \\ \text{or } d &= \frac{v_f^2 - v_i^2}{2a} \dots\dots\dots (1) \end{aligned}$$

From second law of motion,

$$F = ma \dots\dots\dots (2)$$

Multiplying eqns. (1) and (2) we get,

$$\begin{aligned} F d &= ma \left(\frac{v_f^2 - v_i^2}{2a} \right) \\ &= \frac{1}{2} m (v_f^2 - v_i^2) \\ F d &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \end{aligned}$$

The work-energy theorem is applicable for any direction of the force relative to the displacement. For instance, an object with K.E can perform work if it is allowed to push or pull on another object. In this case, the work will be taken as negative and the K.E of the object will decrease. The theorem remains valid even if the force may vary from point to point.

4.6 Define escape velocity. Show that an expression for escape velocity can be expressed as $\sqrt{2gR}$, where R and g denote radius of the Earth and acceleration due to gravity, respectively. Also find its numerical value near the surface of the Earth.

Ans Definition: The initial velocity of an object with which it goes out of the gravitational field, is called escape velocity.

$$v_{\text{esc}} = \sqrt{2gR}$$

Derivation:

Consider a body of mass m , which is to be lifted from the surface of the Earth to infinity. For this purpose, we have to provide initial K.E. which carries it to an infinite distance from the surface of the Earth.

$$\text{Initial K.E.} = \frac{1}{2} m v_{\text{esc}}^2$$

The work done in lifting a body from surface of the Earth to an infinite distance is equal to increase in its potential energy.

$$\begin{aligned} \text{Increase in P.E.} &= 0 - \left(-\frac{GMm}{R} \right) \\ &= \frac{GMm}{R} \end{aligned}$$

where M and R are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial K.E. of the body is equal to the increase in its P.E.

$$\frac{1}{2} m v_{\text{esc}}^2 = \frac{GMm}{R}$$

$$\begin{aligned} v_{\text{esc}}^2 &= \frac{2GM}{R} \\ v_{\text{esc}} &= \sqrt{\frac{2GM}{R}} \dots\dots\dots (1) \end{aligned}$$

When a body is lifted up its weight is equal to gravitational force.

$$w = F = mg$$

$$mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2}$$

Multiplying both sides with R we get

$$\frac{GM}{R} = gR$$

Putting this value in equation (1) we have

$$v_{\text{esc}} = \sqrt{2gR} \text{ --- (2)}$$

This is the formula for escape velocity. It is clear from the formula that escape velocity does not depend upon mass of the body.

Value of Escape velocity:

As $g = 9.8 \text{ m s}^{-2}$, $R = 6.4 \times 10^6 \text{ m}$

$$v_{\text{esc}} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$v_{\text{esc}} = 1.12 \times 10^4$$

$$v_{\text{esc}} = 11.2 \times 10^3 \text{ m s}^{-1}$$

$$v_{\text{esc}} = 11 \text{ km s}^{-1}$$

4.7 Explain the interconversion of potential energy and kinetic energy.

OR

State and Explain law of conservation of energy

Ans Case-1 for freely falling body

Consider a body of mass 'm' at rest, at a height 'h' above the surface of the Earth as shown in figure. We can calculate its total energy at three different positions as follows:

At position A:

The body is at height h from the surface of earth, so its potential energy is given by

$$P.E_A = mgh$$

The body is at rest at this position i.e $v = 0$ hence its kinetic energy is zero.

$$K.E_A = 0$$

$$\text{Total Energy} = E_A = K.E_A + P.E_A \\ = 0 + mgh$$

$$E_A = mgh \text{ --- (1)}$$

When we release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

At position B:

Let us calculate the P.E and K.E at position B when the body has fallen a distance 'x'. Now the height from the ground is (h-x), its potential energy is given by

$$P.E_B = mg(h - x)$$

$$\text{and } K.E_B = \frac{1}{2}mv_B^2 \text{ --- (2)}$$

velocity v_B at position B, can be calculated using the 3rd equation of motion,

$$v_f^2 - v_i^2 = 2aS$$

As $v_i = 0$, $v_f = v_B$, $S = x$ and $a = g$

$$v_B^2 - 0 = 2gx$$

$$v_B^2 = 2gx$$

Therefore, equation (2) becomes

$$K.E_B = \frac{1}{2}m(2gx) = mgx$$

$$\text{Total Energy} = E_B = K.E_B + P.E_B$$

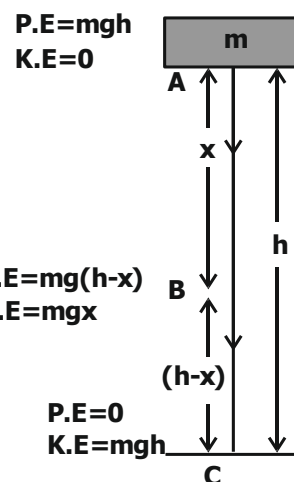
$$E_B = mgx + mg(h - x)$$

$$= mgx + mgh - mgx$$

$$E_B = mgh \text{ --- (2)}$$

At position C:

Just before the body strikes the Earth, its potential energy becomes zero i.e.,



$$P.E_C = 0$$

and K.E. is

$$K.E_C = \frac{1}{2}mv_C^2$$

v_C can be found out by the following expression,

$$v_f^2 - v_i^2 = 2aS$$

As $v_i = 0, v_f = v_C, S = h$ and $a = g$

$$v_C^2 - 0 = 2gh$$

$$v_C^2 = 2gx$$

$$\text{Therefore, } K.E_C = \frac{1}{2}m(2gh) = mgh$$

$$\text{Total Energy} = E_C = K.E_C + P.E_C = mgh + 0$$

$$E_C = mgh \text{ --- (3)}$$

From equations (1), (2) & (3) it is clear that potential energy is converted into kinetic energy, but total energy remains constant.

When body moves in the downward direction its velocity increases and height decreases. Hence it loses P.E. and gains K.E. i.e.,

$$\text{Loss of P.E.} = \text{Gain in K.E.}$$

$$mgh_1 - mgh_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$

This result is only true when frictional force is not considered.

Case-2 Including air friction

If we assume that frictional force 'f' is present during the downward motion, then a part of P.E. is used in doing work against friction and it is equal to 'f h'. The remaining P.E. is converted into K.E. Hence,

$$\text{Loss in P.E.} - \text{work done against friction} = \text{Gain in K.E.}$$

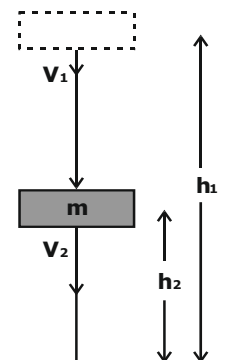
$$mgh - fh = \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2 + fh$$

$$\text{Loss in P.E.} = \text{Gain in K.E.} + \text{Work done against friction}$$

Conversely,

$$\text{Loss of K.E.} = \text{Gain in P.E.} + \text{Work done against friction}$$



Numerical Problems

4.1 A machine gun fires 6 bullets per minute with a velocity of 700 m s^{-1} . If each bullet has a mass of 40 g, then find power developed by the gun?

Given data:

no. of bullets = $N = 6$

$v = 700 \text{ m s}^{-1}$

$m = 40 \text{ g} = 0.040 \text{ kg}$

Total mass of the bullets = $N \times m = 6 \times 0.040 = 0.240 \text{ kg}$

$t = 60 \text{ s}$

To Find: $P = ?$

$$\text{Solution: } P = \frac{W}{t} = \frac{K.E}{t} = \frac{\frac{1}{2}(N \times m) v^2}{t}$$

$$P = \frac{\frac{1}{2} \times 0.240 \times (700)^2}{60}$$

$$P = 980 \text{ W}$$

Ans

4.2 A family uses 10 kW of power. Direct solar energy is incident on horizontal surface at an average rate of 300 W per square meter. If 75% of this energy can be converted into useful electrical energy, how large area is needed to supply 10 kW?

Given data:

$$P = 10 \text{ kW} = 10000 \text{ W}$$

$$I = 300 \text{ W m}^{-2}$$

$$\eta = 75\% = \frac{75}{100} = 0.75$$

To Find: $A = ?$

Solution: Efficiency = $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{A \times I}$ ($\because I = \frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{W}{A \times t} = \frac{P_{\text{in}}}{A}$, $P_{\text{in}} = A \times I$)

$$A = \frac{P_{\text{out}}}{\eta \times I} = \frac{10000}{0.75 \times 300}$$

$$A = \frac{10000}{225} = 44.44 \text{ m}^2 \quad \text{Ans}$$

4.3 The mass of the Earth is $6.0 \times 10^{24} \text{ kg}$ and mass of Sun is $1.99 \times 10^{30} \text{ kg}$. The sun is 160 million km away from the Earth. Find the value of gravitational P.E. of the Earth.

Given data:

$$M_e = 6.0 \times 10^{24} \text{ kg}$$

$$M_s = 1.99 \times 10^{30} \text{ kg}$$

$$r = 160 \text{ million km} = 1.6 \times 10^{11} \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2}$$

To Find: Gravitational P.E = $U = ?$

Solution: $U = -G \frac{M_e M_s}{r}$

$$= -6.67 \times 10^{-11} \frac{6.0 \times 10^{24} \times 1.99 \times 10^{30}}{1.6 \times 10^{11}}$$

$$= - \frac{6.67 \times 6.0 \times 1.99 \times 10^{30+24-11-11}}{1.6}$$

$$= -49.77 \times 10^{32} \text{ J}$$

$$U = -4.97 \times 10^{33} \text{ J} \quad \text{Ans}$$

4.4 An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1} . What is the frictional force acting on it?

Given data:

$$w = 98 \text{ N}$$

$$h = 10 \text{ m}$$

$$v = 12 \text{ m s}^{-1}$$

$$g = 9.8 \text{ m s}^{-2}$$

To Find: $f = ?$

Solution:

$$\text{Here, } m = \frac{w}{g} = \frac{98}{9.8} = 10 \text{ kg}$$

For falling objects,

$$mgh = \frac{1}{2}mv^2 + f \times h$$

$$10 \times 9.8 \times 10 = \left(\frac{1}{2} \times 10 \times 12^2\right) + (f \times 10)$$

$$980 = 5 \times 144 + 10 f$$

$$10 f = 980 - 720 = 260$$

$$f = \frac{260}{10} = 26 \text{ N} \quad \text{Ans}$$

4.5 A 75 watt fan is used for 8 hours daily for 30 days. Find:

(i) energy consumed in electrical units

(ii) electricity bill of fan if one unit cost Rs. 22.5?

Given data:

$$P = 75 \text{ W}$$

$$t = 8 \text{ hours} \times 30 \text{ days} = 240 \text{ hours}$$

To Find:

Energy consumed in electrical units =?

Electricity bill of fan =?

Solution:

(i)

$$P = \frac{\text{Energy consumed}}{t}$$

$$\text{Energy consumed} = P \times t$$

$$= 75 \times 240 \text{ W h} = 18000 \text{ W h}$$

$$\text{As } 1 \text{ electrical unit} = 1 \text{ kW h} = 1000 \text{ W h}$$

$$1 \text{ W h} = \frac{1}{1000} \text{ units}$$

$$\text{So Energy consumed} = 18000 \times \frac{1}{1000} = \mathbf{18 \text{ units}}$$

$$(ii) \text{ Total electricity bill} = \text{energy consumed in units} \times \text{price of 1 unit}$$

$$\text{Total electricity bill} = 18 \times 22.5 = \mathbf{Rs. 405}$$

4.6 If an object of mass 2 kg is thrown up from ground reaches a height of 5 m and falls back to the Earth (neglecting air resistance), calculate:

(i) work done by gravity when object reaches at 5 m height.

(ii) work done by gravity when the object comes back to the Earth.

(iii) Total work done by gravity in upward and downward motion. Also mention physical significance of the result.

Given data:

$$m = 2 \text{ kg}$$

$$h = 5 \text{ m}$$

$$g = 9.8 \text{ m s}^{-2}$$

Solution: Work done by gravity is given by:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta = mgh \cos \theta$$

$$\text{Where } \theta = 0^\circ \quad (\text{for downward motion})$$

$$\theta = 180^\circ \quad (\text{for upward motion})$$

(i) **Work done by gravity when object reaches 5 m (going up)**

$$W = mgh \cos \theta$$

$$W_{\text{up}} = mgh \cos 180^\circ$$

$$= mgh \times (-1) = -mgh \quad (\because \cos 180^\circ = -1)$$

$$= -2 \times 9.8 \times 5$$

$$W_{\text{up}} = \mathbf{-98 \text{ J}} \quad \text{Ans}$$

(ii) **Work done by gravity when object comes back (going down)**

When displacement and gravity are in same direction i.e. $\theta = 0^\circ$, So

$$W_{\text{down}} = mgh \cos 0^\circ$$

$$= mgh \times (1) = mgh \quad (\because \cos 0^\circ = 1)$$

$$= 2 \times 9.8 \times 5$$

$$W_{\text{down}} = \mathbf{98 \text{ J}} \quad \text{Ans}$$

(iii) **Total work done by gravity:**

Total work = work during upward motion + work during downward motion

$$W_{\text{total}} = W_{\text{up}} + W_{\text{down}}$$

$$= -98 + 98$$

$$W_{\text{total}} = \mathbf{0 \text{ J}} \quad \text{Ans}$$

Physical Significance: Work done in a closed path in a conservative field is zero.

4.7 An electrical motor of one horse power is used to run a water pump. Water pump takes 15 minutes to fill a tank of 400 litres at a height of 10 m (1hp = 746 W). Find:

(a) actual input work done by electric motor to fill the tank

(b) actual output work done

Given data:

$$P = 1 \text{ hp} = 746 \text{ W}$$

$$t = 15 \text{ minutes} = 15 \times 60 = 900 \text{ s}$$

$$m = 400 \text{ kg} \quad (400 \text{ litres contain } 400 \text{ kg mass})$$

$$h = 10 \text{ m}$$

To Find:

(a) $W_{\text{Input}} = ?$

(b) $W_{\text{output}} = ?$

Solution:

(a) $P = \frac{W}{t}$

$$W_{\text{input}} = P \times t$$

$$= 746 \times 900 = 671400 \text{ J}$$

$$W_{\text{input}} = \mathbf{671.4 \text{ kJ}} \quad \text{Ans}$$

(b) $W_{\text{outout}} = Fd \cos \theta = mgh \cos(0)$

$$= 400 \times 9.8 \times 10 \times 1 = 39200 \text{ J}$$

$$W_{\text{output}} = \mathbf{39.2 \text{ kJ}} \quad \text{Ans}$$

4.8 A passenger just arrives at the airport and dragging his suitcase to luggage checks in at the desk. He pulls strap with a force of 200 N at an angle of 45° to the floor to displace it 50 m to the desk. Determine the value of work done by him on the suitcase.

Given data:

$$F = 200 \text{ N}$$

$$\theta = 45^\circ$$

$$d = 50 \text{ m}$$

To Find: $W = ?$

Solution:

$$W = Fd \cos \theta$$

$$= 200 \times 50 \times \cos(45^\circ) = 7071 \text{ J}$$

$$W = \mathbf{7 \text{ kJ}} \quad \text{Ans}$$

4.9 A 1200 kg car is running at a speed of 40 km h^{-1} . How much power will be expended by it to accelerate at 2 ms^{-2} ?

Given data:

$$m = 1200 \text{ kg}$$

$$v = 40 \text{ kmh}^{-1} = 40 \left(\frac{1000}{3600} \right) = 11.11 \text{ ms}^{-1}$$

$$a = 2 \text{ ms}^{-2}$$

To Find: $P = ?$

Solution:

$$F = ma = 1200 \times 2 = 2400 \text{ N}$$

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

$$= 2400 \times 11.11 \cos(0^\circ) = 26666.67 \text{ W}$$

$$P = \mathbf{26.67 \text{ kW}} \quad \text{Ans}$$

4.10 A 200 g apple is lifted to 10m and then dropped. What is its velocity when it hits the ground? Assume that 75% of work done in lifting the apple is transferred to K.E by the time it hits the ground.

Given data:

$$m = 200 \text{ g} = 0.2 \text{ kg}$$

$$h = 10 \text{ m}$$

$$\eta = 75\% = 0.75$$

To Find: $v = ?$

Solution:

$$W = Fd \cos \theta = mgh \cos(0^\circ) = 0.2 \times 9.8 \times 10 \times 1$$

$$W = 19.6 \text{ J}$$

$$\text{K.E on hitting ground} = 75\% \times W = 0.75 \times 19.6 = 14.7 \text{ J}$$

$$\text{K.E} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\text{K.E}}{m}} = \sqrt{\frac{2 \times 14.7}{0.2}} = 12.1 \text{ ms}^{-1}$$

$$\mathbf{v = 12.1 \text{ ms}^{-1}} \quad \mathbf{Ans}$$

PHYSICS DEPARTMENT FCC

Chapter No 5

Exercise

Solid and Fluid Dynamics

MCQs

5.1 The region of stress-strain curve which obeys Hooke's law is:

- (a) proportional limit (b) elastic region (c) plastic region (d) yield limit

5.2 Which of the following is more elastic?

- (a) Rubber (b) Wood (c) Sponge (d) Steel

5.3 Which of the following is polymer solid?

- (a) Wool (b) Glass (c) Sodium chloride (d) Copper

5.4 The effect of decrease of pressure with the increase in speed of a fluid in horizontal pipe is:

- (a) Torricelli's effect (b) Bernoulli's effect (c) Venture's effect (d) Doppler's effect

5.5 The pressure will be low when speed of a fluid is:

- (a) zero (b) high (c) low (d) constant

5.6 As per law of fluid friction for steady streamline flow, the friction:

- (a) varies proportionally to velocity of fluid (b) varies inversely proportional to pressure
(c) does not depend on pressure (d) first increases then decreases

5.7 If a stone is submerged in water and it weighs less in water than in air, this phenomenon is due to:

- (a) the reduction of mass in water (b) increase of density in water
(c) buoyant force acting upwards (d) the gravitational force acting upward

5.8 The principle of floatation is a direct application of:

- (a) Pascal's law (b) Bernoulli's principle
(c) Archimedes' principle (d) Newton's third law

5.9 An ideal flow of any fluid must satisfy:

- (a) Pascal's law (b) Bernoulli's equation
(c) Continuity equation only (d) Both (b) and (c)

5.10 The lift force experienced by an aeroplane wings is primarily due to:

- (a) viscosity of air (b) density of air
(c) pressure difference above and below the wing (d) gravitational force

5.11 In medical field, a venture mask, used to deliver a known oxygen concentration to patients operates is based on:

- (a) Newton's third law (b) Archimedes' principle
(c) Pascal's law (d) Bernoulli's principle

5.12 Which of the following is a defining characteristic of a superfluid?

- (a) Zero viscosity (b) Infinite density
(c) Zero temperature (d) Infinite thermal conductivity

Answer Key					
Q.No.	Option	Q.No.	Option	Q.No.	Option
5.1	A	5.5	B	5.9	D
5.2	D	5.6	A	5.10	C
5.3	A	5.7	C	5.11	D
5.4	B	5.8	C	5.12	A

Short Answer Questions

5.1 What is meant by (i) Cohesive force (ii) Viscosity?

Ans Cohesive Force: The attractive intermolecular force between like molecules is called cohesive force. This force is responsible for holding like molecules together.

Example: The strong cohesive forces between water molecules allow them to form a spherical water droplet.

Viscosity: The frictional effect between different layers of flowing fluids is known as viscosity. It measures the force required to slide one layer of liquid over another.

Example: Honey is more viscous than water.

5.2 Differentiate between streamline and turbulent flow of a fluid.

Streamline Flow: The flow is said to be streamline or laminar if every particle that passes a particular point moves along the same path as followed by particles which passed those points earlier.

Example: The water flowing through a pipe.

Turbulent Flow: The irregular or unsteady flow of fluid is called turbulent flow.

Example: The water falling from a mountain.

5.3 How does pressure change with depth in fluids?

Ans Pressure of fluid is directly proportional to its depth.

Pressure is related to depth of fluid by the following relation

$$P = \rho g h$$

$$P \propto h$$

where, h = depth, P = pressure

As depth increases, the pressure of a fluid also increases and vice versa.

5.4 How is variation in pressure related to speed of a fluid?

Ans The pressure is inversely proportional to speed of a fluid.

According to Bernoulli's equation,

$$P + \frac{1}{2} \rho v^2 = \text{constant}$$

It is clear from above equation for a horizontal pipe, where pressure is high the speed will be low.

5.5 How is the flow rate related to the cross-sectional area and velocity of the fluid?

Ans The flow rate of a fluid through a pipe is directly proportional to both the cross-sectional area and its velocity.

$$\text{Flow rate} = A v$$

i.e.

$$\text{Flow rate} \propto A$$

and

$$\text{Flow rate} \propto v$$

5.6 How do you study the variation in velocity of a fluid at different points in a hose with varying diameter?

Ans The velocity of a fluid at different points in a hose varies inversely with diameter. According to equation of continuity,

$$A v = \text{constant} \quad \text{or} \quad A \propto \frac{1}{v}$$

$$\pi \frac{d^2}{4} \propto \frac{1}{v} \quad \text{where } A = \pi \frac{d^2}{4}$$

It is clear from the above equation that when diameter of the hose increases, the velocity of fluid will decrease and vice versa.

5.7 How does an object float or sink according to Archimedes Principle?

Ans Floating and sinking can be explained according to the Archimedes Principle as follows:

Float: An object floats if its weight is equal to or less than the upthrust. The upthrust is always equal to the weight of the fluid displaced by the object.

Sink: An object sinks into fluid if its weight is greater than the upthrust acting on it.

5.8 How does Archimedes reportedly discover the principle that bears his name?

Ans Archimedes observed that when he stepped into the water bathtub, the level of the water in the bathtub rose. From this observation he discovered the principle that bears his name.

5.9 Why standing near fast moving train is dangerous? Explain briefly.

Ans There is danger that he will fall towards the fast-moving train. As the speed of air between person and fast-moving train is high, the pressure will be low. Whereas the speed of the air on the other side of the person is low and pressure will be high. So due to this pressure difference he might fall towards the train.

5.10 What are some potential applications of superfluidity?

Ans Some of the potential applications of superfluidity are:

- (i) As a coolant for high-field magnets
- (ii) Used in advanced particle detectors
- (iii) Used in industry for leak testing
- (iv) Used in the production of electronic and optical products

5.11 Differentiate between stress, strain and Young's Modulus. Write down their SI units.

Ans Stress: The force applied per unit area to produce any change in the shape, volume or length of a body is called stress. Mathematically,

$$\text{Stress } (\sigma) = \frac{\text{Force}}{\text{Area}}$$

Unit: The SI unit of stress (σ) is newton per square meter (N m^{-2}).

Strain: The strain is a measure of deformation of a solid when stress is applied on it.

Unit: It is dimensionless and has **no units**.

Young's Modulus: The tensile stress applied per unit tensile strain is called Young's Modulus.

$$Y = \frac{\text{Tensile Stress}}{\text{Tensile Strain}}$$
$$Y = \frac{F/A}{\Delta L/L}$$

Unit: Its SI unit is N m^{-2} .

Constructed Response Questions

5.1 The ratio stress/strain remains constant for small deformation. What will be effect on this ratio when the deformation made is very large?

Ans For small deformation, there is a linear relationship between stress and strain given as

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

As deformation becomes large the ratio does not remain constant because strain varies nonlinearly with stress. The deformation exceeds elastic limit and material undergoes plastic deformation, so stress/strain ratio is no longer constant.

5.2 When pure water falls on a flat glass plate, it spreads on the plate while the mercury, when falls on the same plate gets converted into small globules. Why?

Ans In case of water, the adhesive forces between water molecules and glass plate are greater than the cohesive forces among water molecules. Consequently, the water spreads on the glass plate.

In case of mercury, the cohesive forces among mercury molecules are greater than the adhesive force between mercury molecules and glass plate. Consequently, the mercury molecules get converted into small globules.

5.3 According to Bernoulli's theorem, the pressure of a fluid should remain uniform in a pipe of uniform radius. But actually, it goes on decreasing. Why is it so?

Ans Bernoulli's theorem is applicable for an ideal fluid, in which mechanical energy is conserved. Therefore, the pressure of the fluid remains uniform.

In real fluids, the energy is lost because of viscosity and frictional effects between fluid and pipe. Due to this, pressure of fluid goes on decreasing in the pipe of uniform radius.

5.4 Why wings of an aeroplane are rounded outward while flattened inward?

Ans According to Bernoulli's theorem, where speed is high pressure will be low.

- The outward rounded wings will increase the speed and decrease the pressure on the upper side of wing.
- The flattened inward wings will decrease speed and increase the pressure on the lower side of wing.
- This pressure difference creates upward lift.

5.5 What is the difference in real fluid, ideal fluid and super fluid? Which of these really exists in the world? Explain.

Ans Real Fluid: It is a fluid that has viscosity and can be compressed.

Ideal Fluid: It is a fluid that does not have viscosity and cannot be compressed. This type of fluid cannot exist practically.

Super Fluid: It is a fluid with zero viscosity i.e., flow is frictionless. They flow without loss of kinetic energy.

Real Existence: Only real fluids exist in the world.

5.6 Why is the study of super fluids important for advancing our knowledge of low temperature physics?

Ans Superfluids like helium-3 and helium-4, help us in advancing our knowledge of low temperature physics such as:

1. To learn more about superconductivity.
2. For use in cooling superconducting magnets, scientific research and medical uses.
3. To be used in industry for leak testing.
4. For the production of electronic and optical products.

PHYSICS DEPARTMENT FCC

Comprehensive Questions

5.1 Explain in detail the classification of solids with respect to atomic arrangements.

Ans 1) Crystalline solids:

Solids which have a regular arrangement of molecules are called crystalline solids. This arrangement is consistent throughout the crystalline structure. Ceramics and metals are examples of crystalline structure as their atoms are arranged in regular pattern.

The techniques to study arrangement of their atoms are:

- i. X-ray Diffraction (XRD)
- ii. Transmission Electron Microscope (TEM)

The atoms arranged in the structure are not static. They vibrate at their average atomic positions whose amplitude changes with temperature. The cohesive force between atoms, molecules or ions in crystalline solids maintain the strict long range order in spite of atomic vibrations.

Melting Point:

For every crystal there is a temperature at which the vibrations become greater that the structure suddenly breaks up, and the solid melts. This temperature is called melting point of the crystalline solid. Some solids with sharp melting points are Quartz, Calcite, Sugar, Mica, diamond etc.

2) Amorphous solids:

Solids having a random arrangement of molecules are called amorphous or glassy solids.

Word amorphous means without form or structure. Therefore, one can say amorphous solids are liquids with the disordered structure frozen in. These solids are also called glassy solids for example ordinary glass.

Due to their random structure, they don't have a fixed melting point like plastic, glass, fused silicon etc.

3) Polymeric solids:

Solids whose structure is in between crystalline and amorphous solids are called polymeric solids. They can be classified as partially or poorly crystalline solids. These solids are of wide range of naturally occurring and synthetic materials. Examples of these solids are polythene, polystyrene and nylon etc. Chemical formula for naturally occurring rubber in pure state is $(C_5H_8)_n$.

Polymerization:

The process of their formation from simple molecules to long chain, three-dimensional structured molecules is called polymerization.

These materials exhibit good strength to weight ratio despite their low specific gravity.

5.2 What is Archimedes' principle? Explain it in detail to find upthrust.

Ans Statement: When an object is totally or partially immersed in a liquid, an upthrust acts on it equal to the weight of the liquid it displaces.

This principle is named after a Greek scientist, Archimedes. He noticed an upward force acting on an object immersed in a liquid. This results in an apparent loss of weight in the object due to upward force called upthrust.

Explanation:

Consider a solid cylinder of cross-sectional area A and height h immersed in a liquid as shown in figure. Let h_1 and h_2 be the depths of the top and bottom faces of the cylinder respectively from the surface of the liquid. Then

$$h_2 - h_1 = h$$

If P_1 and P_2 are the liquid pressures at depths h_1 and h_2 respectively and ρ is its density, then

$$P_1 = \rho gh_1$$

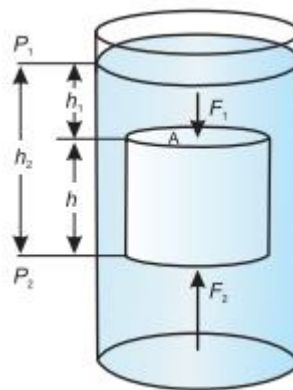
$$P_2 = \rho gh_2$$

Let the force F_1 be exerted at the cylinder top by the liquid due to pressure P_1 and the force F_2 be exerted at the bottom of the cylinder by the liquid due to P_2 , then

$$F_1 = P_1 A = \rho gh_1 A$$

and upthrust

$$F_2 = P_2 A = \rho gh_2 A$$



F_1 and F_2 are the forces acting on the opposite faces of the cylinder. Therefore, net force F will be equal to the difference of these forces. This net force F on the cylinder is called the upthrust of the liquid. Hence

$$\begin{aligned} F_2 - F_1 &= \rho g h_2 A - \rho g h_1 A \\ F_2 - F_1 &= \rho g A (h_2 - h_1) \\ F_2 - F_1 &= \rho g A h \\ \text{Upthrust} &= \rho g V \end{aligned}$$

Here Ah is the volume V of the cylinder and is equal to the volume of the liquid displaced by the cylinder. Therefore, $\rho g V$ is the weight of the liquid displaced. This equation shows that an upthrust acts on a body immersed in a liquid and is equal to the weight of liquid displaced, which is according to Archimedes principle.

5.3 Justify that mass remains conserved when fluid flows through a pipe.

Ans See Answer of Question No. 5.5.

5.4 Explain the term superfluidity.

Ans **Superfluid:** A fluid with zero viscosity is called superfluid. This property of the fluid of being frictionless is called superfluidity.

These fluids flow without loss of kinetic energy.

In the laboratory, superfluids can form in some substances at extremely low temperature. These substances can flow without resistance through extremely narrow passages. They can defy gravity and flow against it in upward direction.

Properties of superfluid:

These substances exhibit unique behavior.

- i. They can climb container walls and escape e.g. helium-3. This phenomenon is called film flow.
- ii. They can pass through container walls.
- iii. On being stirred, superfluids create persistent vortices, unlike regular fluids which settle after some time.
- iv. When the container of the superfluid is rotated, liquid inside remains still.
- v. Some superfluids have high thermal conductivity and varying compressibility.

Example of Superfluid:

- i. Superfluid helium-3
- ii. Superfluid helium-4
- iii. Atomic sodium

Applications:

There are few practical applications currently.

- i. Superfluid helium-4 serves as a coolant for high-field magnets.
- ii. Helium-3 and helium-4 are used in advanced particle detectors.
- iii. It is used for leak testing in industries.
- iv. It is used in production of electronic and optical products.
- v. Liquid helium is recognized for its great thermal conductivity and is used in cryogenic applications, including cooling superconducting magnets, scientific research, and medical uses.

5.5 State and derive equation of continuity.

Ans **Equation of continuity:**

Statement: The product of cross-sectional area of the pipe and the fluid speed at any point along the pipe remains constant. This constant equals the volume flow per second of the fluid or simply flow rate. Mathematically,

$$A v = \text{Constant}$$

Principle: Its working principle is based on the law of conservation of mass.

Proof:

Consider a pipe of non-uniform area.

At Lower End of Pipe:

An ideal fluid enters at the lower end of the pipe and moves a distance Δx_1 with velocity v_1 in time Δt . The area of cross-section of pipe is A_1 . The mass of fluid contained in the shaded region is

$$\text{Density} = \frac{\text{mass}}{\text{Volume}}$$

$$\text{mass} = \text{density} \times \text{Volume}$$

$$\Delta m_1 = \rho_1 A_1 V_1$$

Where ρ_1 is the density of the fluid and V_1 is volume of shaded region Δx_1

as, Volume = Area $\times$ distance

$$V_1 = A_1 \Delta x_1 \quad \text{and} \quad \Delta x_1 = v_1 \Delta t$$

$$\text{So, } \Delta m_1 = \rho_1 A_1 \Delta x_1$$

$$\Delta m_1 = \rho_1 A_1 v_1 \Delta t \text{ ----- (1)}$$

At Upper End of Pipe:

The fluid leaves the pipe at higher end after moving a distance Δx_2 with velocity v_2 during the same time Δt . The area of cross section of the pipe at upper end is A_2 .

The mass of the fluid leaving at the upper end of the pipe is given as

$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t \text{ ----- (2)}$$

For an ideal fluid, the law of conservation of mass can be written as

Mass of fluid entering the pipe = Mass of fluid leaving the pipe

$$\Delta m_1 = \Delta m_2$$

Putting the values from equation (1) and (2), we get

$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

The density of an ideal fluid remains constant, i.e., $\rho_1 = \rho_2 = \rho$

$$A_1 v_1 = A_2 v_2$$

$$Av = \text{Constant}$$

This is equation of continuity.

The constant in equation of continuity is equal to the flow rate (Volume flow per second) of the fluid

i.e., Constant = $\frac{V}{t}$. Therefore,

$$Av = \frac{V}{t} \quad (\because Av = \frac{Ax}{t} = \frac{V}{t})$$

5.6 State and prove Bernoulli's equation.**Ans Bernoulli's Equation**

Statement: The sum of pressure, K.E. per unit volume and P.E. per unit volume of an ideal fluid throughout its steady flow remains constant.

Mathematically,

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{Constant}$$

Principle: It is based on the principle of law of conservation of energy.

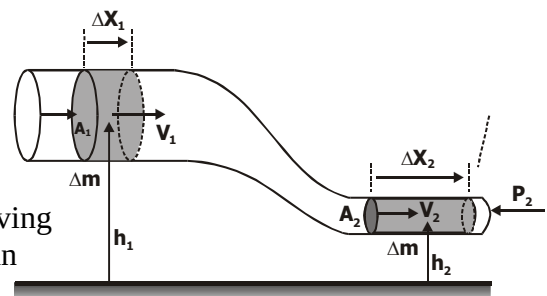
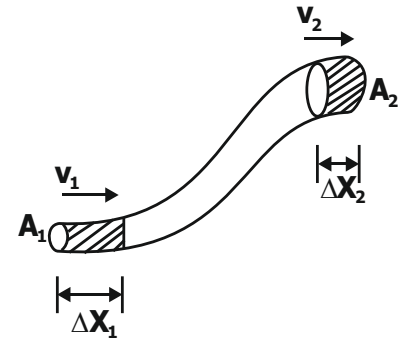
Proof:

Consider an ideal fluid flowing through a closed pipe having different cross-sectional area and height, in time t as shown in figure.

At upper end of pipe:

Let the force on the upper end of the fluid is F_1 , pressure is P_1 and area of cross section is A_1 .

Then,



$$P_1 = \frac{F_1}{A_1}$$

$$F_1 = P_1 A_1$$

The work done on the fluid at the upper end, by the fluid behind it, in moving through a distance Δx_1 is given by

$$W_1 = F_1 \Delta x_1 \cos \theta$$

As force and displacement are in same direction so $\theta = 0^\circ$

$$W_1 = F_1 \Delta x_1 \cos(0^\circ)$$

$$W_1 = P_1 A_1 \Delta x_1 \text{ ----- (1)}$$

At lower end of pipe:

Similarly, the force on the lower end of the fluid is F_2 , pressure is P_2 , area of cross section is A_2 and Δx_2 is the distance moved by the fluid in the same time interval t .

The work done on the fluid at the lower end is given by

$$W_2 = F_2 \Delta x_2 \cos \theta$$

Here force and displacement are in opposite direction hence $\theta = 180^\circ$

$$W_2 = F_2 \Delta x_2 \cos(180^\circ)$$

$$W_2 = - P_2 A_2 \Delta x_2 \text{ ----- (2)}$$

The work W_2 is taken to be negative as this work is done against the fluid force.

Net work done on the fluid can be given as

$$W = W_1 + W_2$$

$$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \text{ ----- (3)}$$

If v_1 and v_2 are velocity of fluid at upper and lower ends respectively, then

$$\Delta x = v_1 t \quad \text{and} \quad \Delta x_2 = v_2 t \quad (\because S = v t)$$

By putting these values in equation (3) we get

$$W = P_1 A_1 v_1 t - P_2 A_2 v_2 t \text{ ----- (4)}$$

From equation of continuity

$$A_1 v_1 = A_2 v_2 = \frac{V}{t} \quad (V = \text{Volume})$$

Multiplying both sides by t we get

$$A_1 v_1 t = A_2 v_2 t = V$$

So equation (4) becomes

$$W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V \text{ ----- (5)}$$

If m is the mass and ρ is density of the fluid then

$$\rho = \frac{m}{V}, \quad V = \frac{m}{\rho}$$

Putting the value of V in equation (5), we have

$$W = (P_1 - P_2) \frac{m}{\rho} \text{ ----- (6)}$$

This work is done by the fluid in changing its K.E. and gravitational P.E.

$$\text{Change in K.E.} = \Delta(\text{K.E.}) = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2$$

$$\text{Change in P.E.} = \Delta(\text{P.E.}) = mgh_2 - mgh_1$$

where h_1 and h_2 are the heights of the upper and lower ends respectively.

Applying law of conservation of energy

$$W = \Delta(\text{K.E.}) + \Delta(\text{P.E.})$$

$$(P_1 - P_2) \frac{m}{\rho} = \left(\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \right) + (mgh_2 - mgh_1)$$

$$(P_1 - P_2) \frac{m}{\rho} = m \left(\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 + gh_2 - gh_1 \right)$$

$$(P_1 - P_2) = \rho \left(\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + gh_2 - gh_1 \right)$$

$$(P_1 - P_2) = \left(\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1 \right)$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

This is Bernoulli's Equation and usually expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$$

5.7 Give some practical applications of Bernoulli's equation.

Ans There are number of applications of Bernoulli's equation.

1. Aeroplane Wings:

The wings of the aeroplane is designed to give it uplift. Its design allow air to pass more swiftly from the upper side of the wing than lower. Hence, the pressure will be lower at the top of the wing. This will provide uplift to the aeroplane.

2. Carburetor

The carburetor of a car engine uses a Venturi duct to feed the correct mixture of air and petrol to the cylinder. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapours into the air stream.

3. Venturi Relation

Consider a pipe within which a fluid of density ρ is flowing through different areas of cross-section as shown in figure. Let A_1 be the cross-sectional area at wide end and A_2 be the cross-sectional area at narrow portion. Suppose that v_1 and v_2 be the flow speeds at the wide and narrow portions respectively. Pressure P_1 and P_2 indicate the liquid pressure at both the portions by connecting the limbs of the manometer. As the pipe is placed horizontally, therefore, we consider that average potential energy is the same at both paces while using Bernoulli's equation.

Thus, Bernoulli's equation can be written as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \dots\dots (1)$$

From the equation of continuity:

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1}$$

As the cross-sectional area A_2 is small as compared to the area A_1 as is clear from the figure, i.e $A_2 < A_1$. So, v_1 will be small as compared to v_2 . Thus, the speed of the fluid is very slow in wider portion of the pipe as compared to the narrow portion. So, we can neglect v_1 on the right-hand side of eqn. (1). Hence,

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2$$

This is known as venturi relation, which is used in venturi meter a device used to measure speed of liquid flow.

4. Torricelli's theorem:

Statement: The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Mathematically, $v_2 = \sqrt{2g(h_1 - h_2)}$

Proof: Suppose a large tank of fluid has two small orifices (holes) A and B on it, as shown in figure. We find the speed v_2 with which fluid flows out of tank from orifice A. As the orifices are so small, the efflux speeds v_2 and v_3 will be much larger than the speed v_1 of the top surface of fluid. Hence v_1 can be approximately taken as zero i.e., $v_1 = 0$.

Applying Bernoulli's equation

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad (\because v_1 = 0)$$

$$P_1 = P_2 = P \text{ (atmospheric pressure)}$$

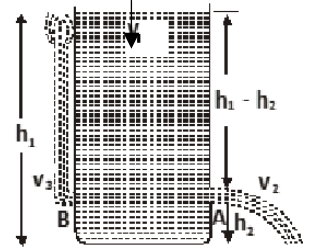
Therefore, above equation becomes

$$\rho g h_1 = \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\frac{1}{2} \rho v_2^2 = \rho g h_1 - \rho g h_2$$

$$v_2^2 = 2g (h_1 - h_2)$$

$$v = \sqrt{2g (h_1 - h_2)}$$



5.8 Define terminal velocity of a body and show that terminal velocity is directly proportional to the square of radius of the body.

Ans: Terminal Velocity: The maximum uniform velocity acquired by a spherical body falling through a fluid when the drag force becomes equal to the weight of the body is called terminal velocity. It is denoted by v_t .

Explanation:

Consider a water droplet having radius r such a fog droplet falling vertically downward through the air. Air exerts an upward drag force on this droplet. As speed of the droplet moving under gravity increases, the drag force on it also increases. When the speed of droplet becomes sufficiently large, the drag force acting on it becomes equal to the weight of the droplet. Hence net force acting on the droplet at this stage becomes zero. Now the droplet moves with maximum uniform velocity called terminal velocity.

Mathematical expression:

$$\text{Net force} = \text{Weight of droplet} - \text{Drag force}$$

$$F_{\text{net}} = w - F_D$$

$$0 = mg - 6\pi\eta r v_t$$

$$6\pi\eta r v_t = mg$$

$$v_t = \frac{mg}{6\pi\eta r}$$

$$v_t = \left(\frac{g}{6\pi\eta r}\right) m \quad \text{--- (1)}$$

$$v_t = \text{constant} \times m \quad \left(\because \frac{g}{6\pi\eta r} = \text{constant}\right)$$

$$v_t \propto m$$

i.e. terminal velocity is directly proportional to the mass of body.

$$\text{as density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{mass} = \text{density} \times \text{volume}$$

$$m = \rho \times \frac{4}{3} \pi r^3$$

$$\left(\because \text{Volume of sphere} = \frac{4}{3} \pi r^3\right)$$

By putting this value in equation (1), we get

$$v_t = \frac{4\pi r^3 \rho g}{3 \times 6\pi\eta r}$$

$$v_t = \frac{2\rho g r^2}{9\eta}$$

$$v_t \propto r^2$$

Thus, terminal velocity is directly proportional to square of the radius of the droplet.

Numerical Problems

5.1 A steel wire of length 2 meters and cross-sectional area $2 \times 10^{-6} \text{ m}^2$ is stretched by a force of 400 N. If the Young's modulus of steel is $2 \times 10^{11} \text{ N m}^{-2}$, calculate the extension of the wire.

Given data:

$$L = 2 \text{ m}$$

$$A = 2 \times 10^{-6} \text{ m}^2$$

$$F = 400 \text{ N}$$

$$Y = 2 \times 10^{11} \text{ N m}^{-2}$$

To find: $\Delta L = ?$

Solution:

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\frac{\Delta L}{L} = \frac{F/A}{Y} = \frac{F}{AY}$$

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L = \frac{400 \times 2}{2 \times 10^{-6} \times 2 \times 10^{11}}$$

$$\Delta L = 0.002 \text{ m} \quad \text{Ans}$$

5.2 A spring with a spring constant 200 N m^{-1} is stretched by 0.5 m. Find the elastic P.E stored in the spring.

Given data:

$$k = 200 \text{ N m}^{-1}$$

$$\Delta x = 0.5 \text{ m}$$

To find: P.E. = ?

Solution:

$$\text{P.E.} = \frac{1}{2} k \Delta x^2$$

$$= \frac{1}{2} (200) (0.5)^2$$

$$\text{P.E.} = 25 \text{ J} \quad \text{Ans}$$

5.3 A copper wire of length 3 meters and cross-sectional area of $1 \times 10^{-6} \text{ m}^2$ is subjected to a force of 500 N. Calculate the stress and strain produced in the wire. (Young's modulus of copper $Y = 1.1 \times 10^{11} \text{ N m}^{-2}$)

Given data:

$$L = 3 \text{ m}$$

$$A = 1 \times 10^{-6} \text{ m}^2$$

$$F = 500 \text{ N}$$

$$Y = 1.1 \times 10^{11} \text{ N m}^{-2}$$

To find: $\sigma = ?$ $\epsilon = ?$

Solution:

$$\sigma = \frac{F}{A} = \frac{500}{1 \times 10^{-6}} = 5 \times 10^8 \text{ N m}^{-2}$$

$$Y = \frac{\sigma}{\epsilon}$$

$$\epsilon = \frac{\sigma}{Y} = \frac{5 \times 10^8}{1.1 \times 10^{11}}$$

$$\epsilon = 0.00455$$

Ans

5.4 A block of wood of mass 10 kg and density of 600 kg m^{-3} is floating in water. Calculate the buoyant force acting on the block. (Density of the water = 1000 kg m^{-3})

Given data:

$$m = 10 \text{ kg}$$

$$\rho = 600 \text{ kg m}^{-3}$$

$$\rho_w = 1000 \text{ kg m}^{-3}$$

To find: $F = ?$
Solution: $F = \rho Vg = mg$ ($\because m = \rho V$)
 $= 10 \times 9.8 = 98 \text{ N}$
F = 98 N **Ans**

5.5: Water flows through a pipe with a diameter of 0.05 m at a velocity of 2 m s^{-1} . If the pipe narrows to a diameter of 0.03 m, calculate the velocity of water at narrow section.

Given data:

$$d_1 = 0.05 \text{ m}$$

$$v_1 = 2 \text{ ms}^{-1}$$

$$d_2 = 0.03 \text{ m}$$

To find: $v_2 = ?$

Solution: $\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{\pi d_1^2 / 4}{\pi d_2^2 / 4} = \frac{d_1^2}{d_2^2}$

$$\frac{v_2}{2} = \frac{0.05^2}{0.03^2}$$

$$v_2 = 5.56 \text{ ms}^{-1}$$
 Ans

5.6 Water flows through a horizontal pipe with a velocity of 3 ms^{-1} and pressure of 200,000 Pa at point 1. At the nozzle (point 2), the pressure decreases to atmospheric pressure 101,300 Pa and the velocity increases to 14 ms^{-1} . Calculate the velocity of water exiting the nozzle. (Density of the water = 1000 kg m^{-3})

Given data:

$$v_1 = 3 \text{ ms}^{-1}$$

$$P_1 = 200,000 \text{ pa}$$

$$P_2 = 101,300 \text{ Pa}$$

To find: $v_2 = ?$

Solution: $\rho = 1000 \text{ kg m}^{-3}$

For horizontal pipe system;

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$200,000 - 101,300 = \frac{1}{2} \times 1000 \times (v_2^2 - 3^2)$$

$$v_2^2 = \frac{2 \times 98700}{1000} + 9 = 206.4$$

$$v_2 = 14.37 \text{ ms}^{-1}$$
 Ans

5.7 A tank filled with water has a hole at a depth of 5 m from the water surface. Calculate the velocity of water flowing out of the hole.

Given data:

$$h_1 = 5 \text{ m}$$

$$h_2 = 0 \text{ m}$$

To find: $v_2 = ?$

Solution: $v_2 = \sqrt{2g(h_1 - h_2)}$

$$v_2 = \sqrt{2 \times 9.8 \times (5 - 0)}$$

$$v_2 = 9.9 \text{ ms}^{-1}$$
 Ans

5.8 Calculate the terminal velocity of a spherical raindrop with a radius 0.5 mm falling through the air. (η for air = $19 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1}$, $\rho = 1000 \text{ kg m}^{-3}$ for water)

Given data:

$$r = 0.5 \text{ mm} = 5.0 \times 10^{-4} \text{ m}$$

$$\rho = 1000 \text{ kg m}^{-3}$$

$$\eta = 19 \times 10^{-6} \text{ kg m}^{-1} \text{ s}^{-1}$$

To find: $v_t = ?$

Solution:

$$v_t = \frac{2gr^2\rho}{9\eta}$$
$$v_t = \frac{2 \times 9.8 \times (5.0 \times 10^{-4})^2 \times 1000}{9 \times 19 \times 10^{-6}}$$
$$v_t = \mathbf{28.65 \text{ m s}^{-1}} \quad \mathbf{Ans}$$

PHYSICS DEPARTMENT FCC

Chapter No 6

Exercise

Heat and Thermodynamics

MCQs

6.1 In an isothermal change, internal energy:

- (A) decreases (B) increases (C) remains the same (D) becomes zero

6.2 First law of thermodynamics is based upon law of conservation of:

- (A) mass (B) energy (C) momentum (D) charge

6.3 A device which converts thermal energy into mechanical energy is called:

- (A) heat engine (B) Carnot Engine (C) refrigerator (D) turbine

6.4 When two objects are made in thermal contact having same temperature, then they are at:

- (A) thermal Equilibrium (B) chemical Equilibrium
(C) mechanical Equilibrium (D) physical Equilibrium

6.5 When the system is expanded by adding heat energy, then the work done will be:

- (A) positive and on the system (B) negative and on the system
(C) positive and by the system (D) negative and by the system

6.6 Entropy of a system in reversible process:

- (A) decreases (B) increases (C) is Infinite (D) is zero

6.7 What happens to internal energy of an object when its temperature:

- (A) decreases (B) remains Constant (C) increases (D) fluctuates

6.8 The value of Boltzmann constant is:

- (A) $1.38 \times 10^{-23} \text{ J K}^{-1}$ (B) $1.38 \times 10^{23} \text{ J K}^{-1}$
(C) $1.38 \times 10^{26} \text{ J K}^{-1}$ (D) $1.38 \times 10^{-23} \text{ J}^{-1} \text{ K}^{-1}$

6.9 In an adiabatic process, there is no:

- (A) change in temperature (B) exchange of heat (C) change in internal energy (D) work done

6.10 Thermodynamics mostly deals with:

- (A) measurement of quantity of heat (B) transfer of quantity of heat
(C) change of state (D) conversion of heat into other forms of energy

Answer Key					
Q.No.	Option	Q.No.	Option	Q.No.	Option
6.1	C	6.5	C	6.9	B
6.2	B	6.6	D	6.10	D
6.3	A	6.7	C		
6.4	A	6.8	A		

Short Answer Questions

6.1 What is meant by thermal equilibrium? Explain briefly.

Ans Thermal Equilibrium: When two bodies are at the same temperature, the thermal energy of each body is equal. As a result, there is no driving force for heat transfer between them. Thus, they remain in thermal equilibrium.

Example: When we put a metal spoon into a cup of hot coffee, the heat flows from coffee to spoon. After some time, both reach at the same temperature, and they are said to be in thermal equilibrium.

6.2 What is meant by internal energy? How is it related to temperature of an ideal gas?

Ans Internal Energy (U):

The sum of all forms of molecular energies (kinetic and potential) of a substance is known as its internal energy.

For an ideal gas, the internal energy is directly proportional to its temperature (T).

$$U \propto T$$

6.3 State 2nd Law of Thermodynamics in two different forms.

Ans i) Lord Kelvin's Statement: It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

ii) Clausius Statement: It is impossible to construct a device that may transfer heat from low temperature reservoir to high temperature reservoir without doing some external work on it.

6.4 Is it possible to construct a heat engine of 100 % efficiency? Explain.

Ans It is impossible to construct a heat engine of 100 % efficiency.

The efficiency of a heat engine depends on the temperature of hot and cold reservoirs. The larger the temperature difference of two reservoirs, the greater is the efficiency. But it can never be one or 100% unless the cold reservoir is at absolute temperature.

6.5 Differentiate between reversible & irreversible processes.

Ans Reversible Process: A succession of events which brings the system back to its initial condition is called a cycle. A reversible cycle is the one in which all the changes are reversible.

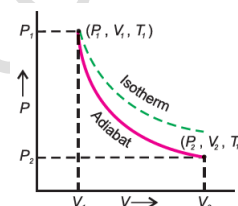
Example: The processes of liquefaction and evaporation of a substance which are performed slowly are practically reversible.

Irreversible process: If a process cannot be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.

Example: An explosion is an example of highly irreversible process.

6.6 Why adiabat is steeper than isotherm? Explain.

Ans Adiabat is steeper than an isotherm because adiabatic process involves no heat exchange leading to a larger temperature change for a given change in volume as compared to the isothermal process in which temperature remains constant.



6.7 A refrigerator transforms heat from cold to hot body. Does this violate the second law of thermodynamics? Justify your answer.

Ans No, it is not a violation of 2nd Law of Thermodynamics.

A refrigerator transfer heat from a low-temperature compartment (cold body) to higher-temperature surroundings (hot body) with the help of external work. It is a heat engine operating in reverse order.

6.8 Explain briefly heat death of universe in terms of entropy.

Ans Heat death of universe is when the universe has reached its maximum value of entropy. This leads to a state where all objects reach thermal equilibrium with their surroundings and no energy is available for useful work. Since no more work can be extracted from the universe at this point, it is dead in effect.

6.9 Is it possible for a cyclic reversible heat engine to absorb heat at constant temperature and transforms it completely into work without rejecting some heat at low temperature? Explain.

Ans No, it is impossible.

According to Kelvin's statement, it is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

6.10 How does behavior of real gases differ from ideal gas at high pressure and low temperature? Identify the reasons behind these differences based on kinetic theory of gases.

Ans Real gasses have intermolecular forces that become significant at high pressure and low temperature, leading to deviations from ideal gas behavior.

At low temperature the gas molecules have lower kinetic energy, and their collision become inelastic, which leads to loss of K.E. and deviation from ideal gas behavior.

6.11 Show that area under P-V graph is equal to work done.

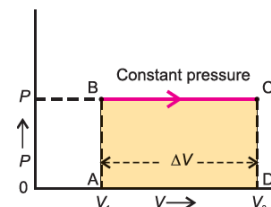
Ans When the gas in a cylinder is heated, it expands steadily through ΔV and the piston moves through a distance Δy . The work done by the gas is

Work done = Area under PV graph

$$W = (AB) \times (AD) \quad (\because A = l \times w)$$

$$W = (P) \times (V_2 - V_1)$$

$$W = P\Delta V$$



6.12 How is work done i) by a gas ii) on a gas?

Ans i) Work done by a gas

When a gas expands and it pushes the piston outward. The work is done by the gas on the surrounding. This work done is taken as positive.

ii) Work done on a gas

When a gas is compressed by applying external force on the piston it moves inward, the work is done on the gas. This work done is taken as negative.

Constructed Response Questions

6.1 Explain how thermodynamics relates to the concept of energy conservation.

Ans The first law of thermodynamics expresses the law of conservation of energy that how energy is transferred within systems. Mathematically, this law can be expressed as

$$\Delta Q = \Delta U + \Delta W$$

Where ΔQ is total amount of heat added to system. ΔU is change in internal energy of system and ΔW is work done by the system.

$$\Delta U = \Delta Q - \Delta W$$

This shows that how energy is converted from one form to another form but the total energy of an isolated system remains conserved.

6.2 Explain how thermodynamics applies to biological systems, such as human body.

Ans Thermodynamics plays an important role in biological systems. Human metabolism provides an example of energy conservation. Human beings do work when they walk, run, or even move. Work requires energy. Work done will result in the decrease in internal energy of the body. Consequently, the body temperature or internal energy is maintained by the food we take. Thus, human metabolism is according to the first law of thermodynamics.

6.3 A gas is expanding adiabatically. Explain what happens to temperature and pressure of the gas.

Ans The temperature and pressure of a gas decrease during an adiabatic expansion.

For an adiabatic process, $Q = 0$. From first law of thermodynamics,

$$Q = \Delta U + W$$

$$0 = \Delta U + W$$

$$\Delta U = -W$$

This shows that during this process, work is done by the gas at the expense of its internal energy due to which the temperature and hence pressure also decreases.

6.4 A coffee cup is left on a table, and overtime coffee cup cools down. Explain thermodynamics processes occurring during this process.

Ans When a coffee cup is left on the table it cools down due to heat transfer. Heat transfers from the hot cup to the surrounding through conduction, convection and radiation. Thus, the temperature of the coffee cup decreases due to heat losses.

6.5 How can we explain different weather patterns through thermodynamical process like wind, rain, etc.

Ans We can explain different weather patterns like wind, rain, etc. through thermodynamic processes involving energy transfer, phase change, pressure difference and laws of thermodynamics as follows:

Wind: When earth is heated by sunlight, then warm air rises and cold air sinks. This creates a pressure difference, and hence wind moves from high to low pressure areas. This wind movement is associated with adiabatic expansion and compression.

Rain: When warm air rises then it expands and cools, this causes condensation of water vapours into droplets. When these droplets become too heavy, they fall to the ground as rain.

Comprehensive Questions

6.1 What are the postulates of kinetic theory of gases? Derive a relation for ideal gas equation in the form $PV = Nk_B T$ from general gas equation.

Ans Kinetic Theory of Gases:

The kinetic theory of gases is a fundamental theory in physics and chemistry that explains the behavior of gases based on the movement of their particles. It helps us to understand large scale properties of gases like pressure, temperature and volume. The postulates of kinetic theory of gases are given below:

1. Gas Particles are in constant, Random Motion:

Gas molecules move constantly in a random motion. They move in straight lines until they collide with each other or with the walls of container.

2. Negligible Volume of Gas Particles:

The volume of each gas molecule is very small as compared to the total volume of the gas. So, these particles are considered as point masses with no volume.

3. No Intermolecular Forces:

There are no attractive or repulsive forces between molecules. They do not exert force on each other except during collisions.

4. Elastic Collisions:

Collisions between gas molecules and with walls of the container are perfectly elastic. It means the total kinetic energy during the collisions, is conserved.

5. Large Number of Particles:

A gas contains a large number of particles. The large number of particles allows statistical methods to be used to understand the gas properties.

6. Average kinetic Energy is Proportional to Temperature:

The average kinetic energy of gas is directly proportional to the absolute temperature of a gas. As the temperature rises, the particles move faster.

7. Pressure due to Particle Collisions:

Pressure is exerted by gas particles colliding with the walls of the container. These collisions exert force which produces pressure.

8. Time of collisions is Negligible:

The time interval of a collision between gas particles is very short compared to the time between collisions. This assumption makes it easier to study particle dynamics.

Derivation of a relation for ideal gas equation, $PV = Nk_B T$, from general gas equation:

Equation of State for Ideal Gas:

A gas that obeys kinetic theory of gases is termed as an ideal gas. Ideal gas equation is given by

$$PV = nRT \text{ ----- 1}$$

Here P = pressure, V = volume, n = number of moles of the gas, R = universal gas constant ($R = 8.3145 \text{ J mol}^{-1} \text{ K}^{-1}$) and T = absolute temperature.

Equation 1 shows that product of pressure and volume is directly proportional to the absolute temperature for an ideal gas.

Real Gas to Behave Like an Ideal Gas:

According to kinetic theory of gases, a gas has no intermolecular interaction and molecules are far apart from each other. For a real gas to behave like an ideal gas, some conditions must be satisfied. P.E. of the gas molecules is negligible and they have only K.E.

Number of moles 'n' can be given by

$$n = \frac{\text{Mass of gas}}{\text{Molar mass of gas}} = \frac{m}{M}$$

Putting in Equation 1

$$\begin{aligned} PV &= \frac{m}{M} RT \\ PM &= \frac{m}{V} RT & (\because \rho = \frac{m}{V}) \\ PM &= \rho RT \\ \rho &= \frac{PM}{RT} \\ \rho &\propto \frac{P}{T} & (\because \frac{M}{R} = \text{constant}) \end{aligned}$$

The density of a gas will be low at low pressure and high temperature, due to which molecules of the gas will be at large distance from each other and the intermolecular forces will be negligible. So, real gas behaves like an ideal gas at low pressure and high temperature.

Ideal Gas Equation in the form $PV = Nk_B T$, (in terms of Boltzmann Constant)

Considering ideal gas equation,

$$PV = nRT \text{ -----1}$$

where n represents number of moles of the ideal gas.

A mole can be defined as the number of atoms or molecules per unit Avogadro's number
 $(N_A = 6.02 \times 10^{23})$

$$\text{i.e., } n = \frac{N}{N_A}$$

Equation (1) becomes

$$PV = \frac{N}{N_A} RT$$

$$= \frac{R}{N_A} NT$$

The term $\frac{R}{N_A}$ is termed as Boltzmann Constant k_B i.e.,

$$PV = Nk_B T \text{ -----2}$$

Equation 2 gives ideal gas equation in terms of Boltzmann Constant k_B .

6.2 State and explain various gas laws.

Ans Gas Laws:

The physical Laws which describe the mutual relationship between pressure, volume and temperature for an ideal gas are termed as gas laws. In these laws, if one variable changes, the second variable also changes while the third remains constant.

Boyle's Law:

This law was introduced by Robert Boyle in 1662 and it provides a relationship between pressure and volume of a gas at constant temperature.

It states that for a fixed mass of an ideal gas, the pressure P exerted by a gas varies inversely with volume V occupied by the gas at constant temperature.

Mathematically;

$$P \propto \frac{1}{V}$$

at constant T

$$P = \text{constant} \frac{1}{V} \quad \text{or}$$

$$PV = \text{constant}$$

$$P_1 V_1 = P_2 V_2$$

Graphical representation of Boyle's law is shown in fig 1.

Charle's Law

This law was formulated in 1870, by a French Physicist Jacques Charles. It provides a relationship between volume and temperature of an ideal gas at constant pressure.

It states that the volume of given mass of gas is directly proportional to the absolute temperature at constant pressure. Mathematically;

$$V \propto T \quad \text{at constant P}$$

$$\frac{V}{T} = \text{constant}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Graphical representation of Charle's Law is shown in fig. 2.

Joseph Lussac's Law

It states that for a fixed mass of an ideal gas, the pressure exerted by a gas varies directly with absolute temperature of the gas at constant volume.

Mathematically;

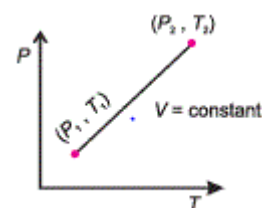
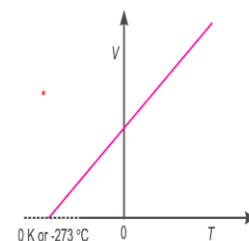
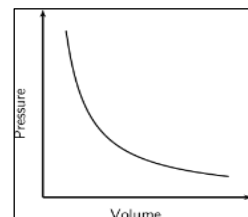
$$P \propto T \quad \text{at constant V}$$

$$P = \text{constant} \times T$$

$$\frac{P}{T} = \text{constant}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Graphical representation of Joseph Lussac's Law is shown in fig.



6.3 Explain first law of thermodynamics in detail. Give an example in support of your explanation. Give its two applications.

Ans 1st law of Thermodynamics:

In any thermodynamic process, when heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus work done W by the system on its surroundings.

Explanation:

When heat is added to a system, there is an increase in the internal energy ΔU , due to the rise in temperature, an increase in pressure or change in the state. If at the same time, a substance is allowed to expand by doing work on its environment, then the heat Q required will be:

$$Q = (U_2 - U_1) + W$$

$$Q = \Delta U + W$$

Thus the change in internal energy

$$\Delta U = Q - W$$

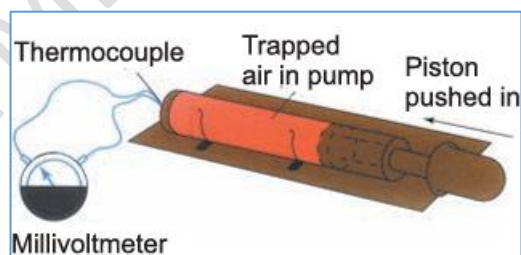
Conservation Principle: The first law of thermodynamics is based on energy conservation. It means that energy can change from one form to another but the total energy in an isolated system remains constant over time.

Wider Applicability: The first law of thermodynamics applies universally to all types of energies and processes including chemical reactions, electrical systems and nuclear reactions. It helps scientists and engineers to understand and predict how energy changes in different situations.

The first law of thermodynamics expresses that energy is always conserved in an isolated system. It shows us that energy can neither be created nor be destroyed.

Examples:

A bicycle pump: When we pump on the handle rapidly, it becomes hot due to mechanical work done on the gas, increasing its internal energy. A typical bicycle pump consists of a blocked outlet as shown in figure. A thermocouple connected through the blocked outlet allows the air temperature to be monitored. Thermocouple thermometer can detect a minute change of the temperature.



When the piston is rapidly pushed, thermometer shows a temperature rise due to increase in internal energy of the air. The push force does work on the air, thus increasing its internal energy, which is shown by the increase in temperature of the air.

Human metabolism: Human beings and other animals do work when they walk, run, or move. Work requires energy, which is also needed for growth to make new cells and to replace old dead cells. Energy transforming processes that occur within an organism are named as metabolism. First law of thermodynamics ($\Delta U = Q - W$), can be applied to an organism of the human body. Work done will result in the decrease in internal energy of the body. As a result, the body temperature or internal energy is maintained by the food we eat. **(Note: Learn any one example.)**

Applications:

1. Isothermal process:

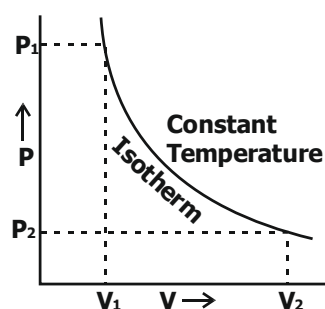
It is a process which is carried out at constant temperature and hence the condition for the application of Boyle's law is fulfilled.

Explanation:

When a gas expands or compresses isothermally, the product of its pressure and volume during the process remains constant. If pressure of the system changes from P_1 to P_2 and volume changes from V_1 to V_2 during an isothermal process, then

$$P_1 V_1 = P_2 V_2$$

or $P V = \text{constant}$



For an ideal gas P.E of gas molecules is zero. So, internal energy of gas molecules depends only on the temperature. As temperature remains constant, hence change in internal energy is zero i.e. $\Delta U = 0$.

Applying 1st Law of thermodynamics,

$$Q = \Delta U + W$$

$$\begin{aligned} Q &= 0 + W \\ Q &= W \end{aligned}$$

(a) Isothermal Expansion:

When gas is allowed to expand at constant temperature, work is done by the system on the surrounding and this is called isothermal expansion. In this case, heat is supplied to the system in order to keep the temperature constant.

$$Q = W$$

(a) Isothermal Compression:

When gas is compressed at constant temperature then work is done on the system, and this is called isothermal expansion. In this case, heat is taken out from the system in order to maintain the temperature.

$$Q = -W$$

Since transfer of heat requires time, hence, to keep the temperature constant, the expansion or compression of gas must take place slowly.

When pressure is plotted against volume for this process, we get a curve as shown in figure. This is called an isotherm.

2. Adiabatic process:

The process in which no heat enters or leaves the system is called the adiabatic process.

In this process entropy of the system remains constant.

Explanation:

As $Q = 0$

Applying 1st Law of Thermodynamics

$$\begin{aligned} Q &= \Delta U + W \\ \Delta U + W &= 0 \end{aligned}$$

(a) Adiabatic Expansion:

When gas is allowed to expand by decreasing the pressure, the work is done by the system at the cost of internal energy. Therefore, internal energy of the system decreases. This is called adiabatic expansion.

$$W = -\Delta U$$

(b) Adiabatic Compression:

When gas is compressed by increasing the pressure, the work is done on the system. Therefore, internal energy of the system increases. This is called adiabatic compression.

$$\Delta U = -W$$

Adiabatic change occurs when the gas expands or is compressed rapidly.

Examples of Adiabatic process:

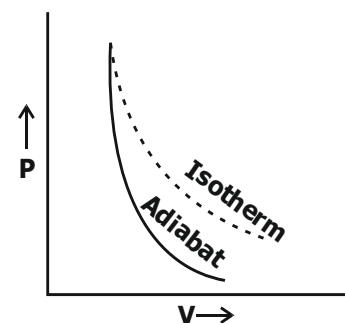
- Rapid escape of air from a burst tyre.
- The rapid expansion and compression of air when sound waves pass through it.
- Formation of clouds in the atmosphere.

As the temperature of the gas does not remain constant, so

$$P V^\gamma = \text{constant}$$

$$\text{where } \gamma = \frac{C_p}{C_v}$$

The curve between P and V for an adiabatic process is called adiabat. It is steeper than an isotherm.



6.4 What is a refrigerator? Explain its working. Derive an expression for its co-efficient of performance.

Ans Refrigerator is a device which maintains the temperature of the body below that of its surrounding.

It operates in a cyclic process but in reverse as that of the heat engine as shown in fig. A refrigerator absorbs heat from a cold reservoir and gives it off to a hot reservoir. This shows that in refrigerator, the work is done on the system while in a heat engine, work is done by the system.

A refrigerator works on the basis of Clausius statement of second law of thermodynamics, i.e., a heat engine is operating in reverse. Heat Q_c is drawn from Low Temperature Reservoir (LTR) T_1 by compressor and is thrown into High Temperature Reservoir (HTR) T_2 with the help of external work done. The heat rejected to HTR (Q_H) is given by

$$Q_c + W = Q_H$$

$$W = Q_H - Q_c$$

The main purpose of refrigerator is to extract as much heat Q_c as possible from LTR with the dissipation of as little work W as possible.

Co-efficient of Performance of Refrigerator:

The ratio of heat removed from LTR (Q_c) to the work done (W) is called co-efficient of performance of a refrigerator.

A better refrigerator will remove a greater amount of heat from inside the refrigerator for the expenditure of a smaller mechanical work or electrical energy.

The co-efficient of performance of a refrigerator can be given by

$$E = \frac{Q_c}{W} = \frac{Q_c}{Q_H - Q_c}$$

The coefficient of performance in terms of temperature, where $Q \propto T$ is:

$$E = \frac{T_1}{T_2 - T_1}$$

6.5 What is Carnot engine? Describe Carnot cycle. State Carnot theorem and derive an expression for the efficiency of Carnot engine.

Ans Carnot Engine:

Sadi Carnot in 1824 described an ideal heat engine using only isothermal and adiabatic processes. He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures would be the most efficient engine.

Carnot Cycle:

A Carnot Cycle using an ideal gas as the working substance is shown in PV-diagram. It consists of following four steps:

1 Isothermal Expansion:

The gas is allowed to expand isothermally at temperature T_1 , absorbing heat Q_1 from the hot reservoir. The process is represented by curve AB.

2 Adiabatic Expansion:

The gas is then allowed to expand adiabatically until its temperature drops to T_2 . The process is represented by curve BC.

3 Isothermal Compression:

The gas at this stage is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservoir. The process is represented by curve CD.

4 Adiabatic Compression:

Finally, the gas is compressed adiabatically to restore its initial state at temperature T_1 . The process is represented by curve DA.

Each process is perfectly reversible because thermal and mechanical equilibrium is maintained all the time.

Carnot's Theorem:

Statement: No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

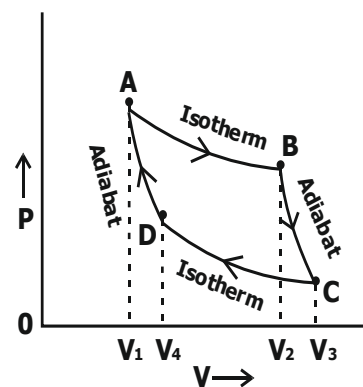
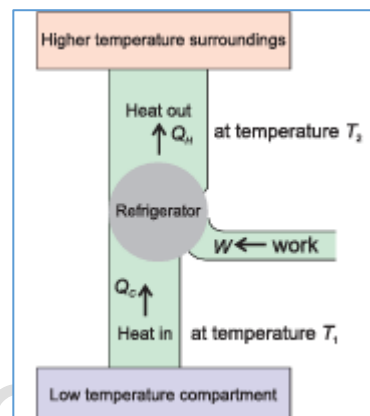
Extended Carnot's Theorem:

Statement: All Carnot engines operating between the same two temperatures have the same efficiency irrespective of the nature of the working substance.

Expression for Efficiency of Carnot Engine:

As the working substance returns to the initial state, there is no change in its internal energy i.e., $\Delta U = 0$.

Net work done:



The net work done during one cycle is equal to the area enclosed by the path ABCDA in the PV – diagram.

$$\begin{aligned}\text{Heat absorbed} &= Q_1 \\ \text{Heat rejected} &= Q_2 \\ \text{Net heat absorbed} &= Q = Q_1 - Q_2\end{aligned}$$

From the 1st Law of thermodynamics

$$\begin{aligned}Q &= \Delta U + W & (\because \Delta U = 0) \\ Q_1 - Q_2 &= 0 + W \\ W &= Q_1 - Q_2\end{aligned}$$

Efficiency (η):

The efficiency is defined as

$$\begin{aligned}\eta &= \frac{\text{Output (work)}}{\text{Input (Energy)}} \\ &= \frac{W}{Q_1} \\ &= \frac{Q_1 - Q_2}{Q_1} \\ \eta &= 1 - \frac{Q_2}{Q_1} \quad \text{----- (1)}\end{aligned}$$

As heat Q_1 and Q_2 is proportional to Kelvin temperatures T_1 and T_2 respectively.

Therefore, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$. Putting in eq. (1)

$$\eta = 1 - \frac{T_2}{T_1} \quad \text{----- (2)}$$

$$\text{Percentage Efficiency} = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

Thus, the efficiency of Carnot engine depends upon the temperatures T_1 and T_2 and not on the nature of working substance. The larger the temperature difference of two reservoirs, the greater is the efficiency. But it can never be 1 or 100% unless cold reservoir is at absolute zero temperature ($T_2 = 0$ K).

$$\begin{aligned}\eta &= 1 - \frac{T_2}{T_1} \\ \eta &= 1 - \frac{0}{T_1} \\ &= 1 - 0 \\ \eta &= 100\%\end{aligned}$$

It is against 2nd law of thermodynamics so efficiency cannot be 100%. It is always less than 1.

The Carnot cycle sets up an upper limit on the efficiency of all heat engines. No practical heat engine can be perfectly reversible and also energy dissipation is necessary.

6.6 Define and explain the term “Entropy”.

Ans Entropy: Entropy is the measure of disorder of a system. It is denoted by S. **OR**

The unavailability of heat energy to do useful work is called entropy.

Explanation:

Entropy is a state variable used to describe the state of a system like temperature, pressure, volume and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of heat ΔQ at absolute temperature T , then the increase in the entropy (S) of the system is given by

$$\Delta S = \frac{\Delta Q}{T}$$

Like potential energy or internal energy, it is the change in entropy (ΔS) of the system which is important. Change in entropy is positive when heat is added and negative when heat is removed from the system.

S.I. Units: J K^{-1}

2nd law of thermodynamics in terms of entropy:

Statement: If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

Explanation:

Suppose Q amount of heat flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 and $T_1 > T_2$. The change in entropy of the reservoir, at T_1 , which loses heat, decreases by $\frac{Q}{T_1}$ and of the reservoir at temperature T_2 , which gains heat, increases by $\frac{Q}{T_2}$. As $T_1 > T_2$, thus $\frac{Q}{T_2} > \frac{Q}{T_1}$.

Hence, net change in entropy = $\frac{Q}{T_2} - \frac{Q}{T_1}$ is positive.

It follows that in natural processes where heat flows from one system to another there is always a net increase in the entropy. This is another statement of 2nd law of thermodynamics. It states that "If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase."

It is observed that a natural process proceeds towards a state of greater disorder. So, there is a relation between entropy and molecular disorder.

Example:

1. In an irreversible heat flow from hot to cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of the increase in random motion of molecules. Thus, entropy is increased.
2. Free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus, entropy is increased.

Conclusion:

1. The entropy of a system remains constant for a reversible process.
 2. The entropy of a system increases for all irreversible processes.
- Whenever entropy increases, we cannot convert heat into useful work easily. For example, mixing of hot and cold water means we cannot separate them again. Nothing is lost, but the energy is less useful. In other words, as entropy increases, energy goes from a more organized form to a less organized form, and it eventually turns into thermal energy.

In all real processes with heat transfer, the energy available for useful work decreases, which means entropy increases. Even if one system's entropy decreases, the overall entropy of the universe still increases because another system's entropy increases more. When we consider all systems together, the entropy of the universe always increase

Numerical Problems

- 6.1 A gas occupies 6.0 L of volume at a pressure of 12 atm. What will be the volume of gas if the pressure is increased by 2.0 atm, assuming that temperature remains constant?**

Given data:

Initial Volume = $V_1 = 6.0 \text{ L}$
Initial Pressure = $P_1 = 12 \text{ atm}$
Final Pressure = $P_2 = 12 + 2 \text{ atm} = 14 \text{ atm}$
Temperature = $T_1 = T_2 = T = \text{constant}$

To find:

Final Volume = $V_2 = ?$

Solution:

Using Boyle's Law,
 $P_1 V_1 = P_2 V_2$
 $V_2 = \frac{P_1 V_1}{P_2}$

$$= \frac{12 \times 6}{14}$$

$$V_2 = 5.14 \text{ L} \quad \text{Ans}$$

6.2 In a vacuum chamber which is connected to a cryogenic pump, pressure as low as 1.00 nPa is being attained. Calculate the number of molecules in 1.00 m³ vessel at this pressure and temperature of 300 K.

Given data:

$$\text{Pressure} = P = 1.00 \text{ nPa} = 1.00 \times 10^{-9} \text{ Pa}$$

$$\text{Volume} = V = 1.00 \text{ m}^3$$

$$\text{Temperature} = T = 300 \text{ K}$$

$$\text{Boltzmann Constant} = k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

To find:

$$\text{Number of molecules} = N = ?$$

Solution:

According to Ideal Gas Equation,

$$P V = N k_B T$$

$$N = \frac{P V}{k_B T}$$

$$= \frac{1.00 \times 10^{-9} \times 1.00}{1.38 \times 10^{-23} \times 300}$$

$$N = 2.415 \times 10^{11} \text{ molecules} \quad \text{Ans}$$

6.3 A gas undergoes a thermodynamic process where it absorbs 500 J of heat energy and performs 300 J work on its surroundings. Calculate the change in internal energy of the gas.

Given data:

$$\text{Heat absorbed} = Q = 500 \text{ J}$$

$$\text{Work done by gas} = W = 300 \text{ J}$$

To find:

$$\text{Change in internal energy} = \Delta U = ?$$

Solution:

According to 1st Law of Thermodynamics,

$$Q = \Delta U + W$$

$$\Delta U = Q - W$$

$$= 500 - 300 \text{ J}$$

$$\Delta U = 200 \text{ J} \quad \text{Ans}$$

6.4 A Carnot engine is operating between a high temperature reservoir at 600 K and a low temperature reservoir at 300 K. Calculate:

i) the maximum possible efficiency

ii) the amount of work output if the engine absorbs 500 J of heat from the high temperature reservoir.

Given data:

$$T_1 = 600 \text{ K}$$

$$T_2 = 300 \text{ K}$$

$$Q_1 = 500 \text{ J}$$

To find:

$$\text{i) } \eta = ?$$

$$\text{ii) } W = ?$$

Solution:

i) We know that,

$$\eta = \left[1 - \frac{T_2}{T_1} \right] \times 100 \%$$

$$= \left[1 - \frac{300}{600} \right] \times 100 \%$$

$$\eta = 50 \% \quad \text{Ans}$$

ii) We know that,

$$\eta = \frac{W}{Q_1}$$

$$W = \eta \times Q_1$$

$$= 0.5 \times 500$$

$$W = 250 \text{ J} \quad \text{Ans}$$

6.5 A refrigerator extracts 1200 J of heat from its interior (the cold reservoir) and releases 1800 J of heat to the surrounding environment (the hot reservoir) during each cycle. Calculate:

- i) the work input required per cycle.
 ii) the coefficient of performance (COP) of the refrigerator.

Given data:

$$Q_C = 1200 \text{ J}$$

$$Q_H = 1800 \text{ J}$$

To find:

- i) $W = ?$
 ii) $E = ?$

Solution:

i) We know that,

$$Q_H = Q_C + W$$

$$W = Q_H - Q_C$$

$$= 1800 - 1200 \text{ J}$$

$$W = 600 \text{ J} \quad \text{Ans}$$

ii) We know that,

$$E = \frac{Q_C}{W}$$

$$= \frac{1200}{600}$$

$$E = 2 \quad \text{Ans}$$

6.6 Calculate the entropy change when 1.0 mole of ice at 0°C melts to form liquid water at the same temperature. (Heat of fusion of ice per mole = $6.01 \times 10^3 \text{ J}$)

Given data:

$$n = 1.0 \text{ mole}$$

$$T = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}$$

$$\Delta Q_{\text{fusion}} = 6.01 \times 10^3 \text{ J} = 6010 \text{ J}$$

To find:

$$\Delta S = ?$$

Solution:

We know that,

$$\Delta S = \frac{\Delta Q_{\text{fusion}}}{T}$$

$$= \frac{6010}{273}$$

$$\Delta S = 22.0 \text{ J K}^{-1} \quad \text{Ans}$$

6.7 A gas occupies 400 mL at 20°C . What volume will it occupy at 80°C , assuming constant pressure?

Given data:

$$V_1 = 400 \text{ mL}$$

$$T_1 = 20^\circ\text{C} = 20 + 273 = 293 \text{ K}$$

$$T_2 = 80^\circ\text{C} = 80 + 273 = 353 \text{ K}$$

$$P = \text{constant}$$

To find:

$$V_2 = ?$$

Solution:

Using Charles Law,

$$\begin{aligned}\frac{V_1}{T_1} &= \frac{V_2}{T_2} \\ V_2 &= \frac{V_1 T_2}{T_1} \\ &= \frac{400 \times 353}{293}\end{aligned}$$

$$V_2 = 482 \text{ mL}$$

Ans

6.8 A gas has a pressure of 2 atm at 300 K. What pressure will it have at 450 K, assuming constant volume?

Given data:

$$\begin{aligned}P_1 &= 2 \text{ atm} \\ T_1 &= 300 \text{ K} \\ T_2 &= 450 \text{ K}\end{aligned}$$

To find:

$$P_2 = ?$$

Solution:

Using Gay-Lussac's Law,

$$\begin{aligned}\frac{P_1}{T_1} &= \frac{P_2}{T_2} \\ P_2 &= \frac{P_1 T_2}{T_1} \\ &= \frac{2.0 \times 450}{300} \\ P_2 &= 3.0 \text{ atm}\end{aligned}$$

Ans

Chapter No 7

Waves and Vibrations

Exercise

MCQs

7.1 The simple wave speed equation is represented by:

- (a) $v = f\lambda$ (b) $v = \frac{s}{t}$ (c) $v = r\omega$ (d) $v = \frac{\Delta d}{\Delta t}$

7.2 The principle of superposition in waves is stated as:

- (a) the displacement of a wave is the sum of the displacements of its individual components
(b) the velocity of a wave is the product of its individual components
(c) the frequency of a wave is the difference of its individual components
(d) the amplitude of a wave is the ratio of its individual components

7.3 A node in a stationary wave is:

- (a) a point of maximum displacement (b) a point of intermediate displacement
(c) a point of zero displacement (d) a point of infinite displacement

7.4 An antinode in a stationary wave is:

- (a) a point of maximum displacement (b) a point of minimum displacement
(c) a point of zero displacement (d) a point of infinite displacement

7.5 Stationary waves are defined as:

- (a) waves that move with a constant velocity (b) waves that move with a changing velocity
(c) waves that oscillate in a fixed position (d) waves that propagate through a medium

7.6 Harmonics are:

- (a) integer multiples of a fundamental frequency (b) integer submultiples of a fundamental frequency
(c) random frequencies (d) non-integer multiples of a fundamental frequency

7.7 The result of constructive interference between two waves is represented as:

- (a) a decrease in amplitude (b) an increase in amplitude
(c) no change in amplitude (d) a shift in phase

7.8 If the amplitude of the wave is tripled, then the amount of energy is increased by:

- (a) 3 times (b) 6 times (c) 9 times (d) 12 times

7.9 What type of waves do headphones use to produce sound?

- (a) Electromagnetic waves (b) Mechanical waves
(c) Pressure waves (d) Longitudinal waves

7.10 The typical frequency range of microwaves is

- (a) $10^3 - 10^5$ Hz (b) $10^5 - 10^7$ Hz (c) $10^7 - 10^9$ Hz (d) $10^9 - 10^{11}$ Hz

7.11 The bending of waves around an obstacle is called as:

- (a) refraction (b) reflection (c) diffraction (d) interference

7.12 The Doppler Effect used in astronomy is for:

- (a) measuring the diameters of stars (b) determining velocity of galaxies
(c) analyzing properties of black holes (d) studying behavior of electromagnetic waves

Answer Key

Q.No.	Option	Q.No.	Option	Q.No.	Option
7.1	A	7.5	C	7.9	D
7.2	A	7.6	A	7.10	D
7.3	C	7.7	B	7.11	C
7.4	A	7.8	C	7.12	B

Short Answer Questions

7.1 What are the conditions for interference to occur?

Ans Interference of two waves travelling in the same medium occurs when the following conditions are satisfied:

- i) Waves must have same frequency.
 ii) Waves are travelling along same direction (coherent).

7.2 Differentiate between constructive and destructive interference of waves.

Ans Constructive interference: Whenever the path difference is an integral multiple of wavelength, the two waves are added up. This effect is called constructive interference.

Mathematically; $\Delta S = n\lambda$, where $n = 0, \pm 1, \pm 2, \pm 3 \dots$

Destructive interference: Whenever the path difference between the waves is an odd integral multiple of half wavelength, the waves cancel the effect of each other. This effect is called destructive interference

Mathematically, $\Delta S = (2n + 1)\frac{\lambda}{2}$, where $n = 0, \pm 1, \pm 2, \pm 3 \dots$

7.3 What are coherent waves and coherent sources? Give examples.

Ans Coherent waves: Coherent waves are those which have the same frequency and maintain a constant phase difference.

Example: Sound waves emitted by the two loud speakers attached with same audio source.

Coherent sources: The sources producing coherent waves are called coherent sources.

Example: Two Loudspeakers connected with an audio generator producing coherent waves, can act as coherent sources of sound waves.

7.4 Distinguish between longitudinal and transverse waves.

Ans Longitudinal Waves: The waves in which particles of the medium vibrate parallel to the direction of propagation of the waves are called longitudinal waves **e.g.** sound waves.

Transverse waves: The waves in which particles of the medium vibrate perpendicular to the direction of propagation of the waves are called transverse waves **e.g.** waves produced in a string.

7.5 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave? How is it so?

Ans No, it is not possible.

Stationary waves can only be produced when two identical waves of the same frequency travelling in opposite directions, superpose each other.

7.6 How would you apply Doppler effect in studying cardiac problems in humans?

Ans Doppler echocardiography: It measures the blood flow velocity and detects cardiac abnormalities such as valve stenosis or regurgitation. It also helps with pacemaker settings.

Similarly, Doppler Ultrasound measures blood flow and calculates cardiac problems.

7.7 What is meant by diffraction of waves? For what purpose, the ripple tank is used?

Ans Diffraction of waves: It is the bending of waves around the sharp edges or corners of obstacles or spreading of waves beyond a barrier (a slit).

Ripple tank: It is very useful apparatus not only to generate water waves, but also to demonstrate the waves properties like reflection, diffraction and refraction.

Constructed Response Questions

7.1 Which measurement of a wave is the most important when determining the wave's intensity?

Ans The amplitude of wave is the most important factor for measuring the intensity of wave.

The intensity of a wave is directly proportional to the square of its amplitude. So, a wave with greater amplitude carries more energy.

$$I \propto A^2$$

7.2 Can you apply Doppler effect to light waves? Describe briefly.

Ans Yes, the Doppler effect can be applied to light waves.

When light source moves towards an observer, then light waves become shorter in wavelength and spectrum appears to be shifted toward shorter wavelengths end of spectrum i.e. blue end of spectrum. When light source moves away, then light waves are longer in wavelength and spectrum appears to be shifted towards longer wavelength end of spectrum. i.e. red end of spectrum.

7.3 Can you compare the compressions and rarefactions of the longitudinal wave with peaks and troughs of the transverse wave? Discuss.

Ans Yes, we can compare both compressions and rarefactions.

The compressions of the longitudinal waves are the regions where particles are closer together and rarefactions are the regions where they are far apart.

The peaks (crests) of transverse waves are the highest points and troughs are the lowest points.

The compression of longitudinal waves and peak of transverse waves can be compared in terms of maximum value of disturbance. Similarly, the rarefaction and trough can be compared in terms of minimum value of disturbance.

7.4 How should a source of sound move with respect to an observer so that the frequency of its sound does not change? Write two examples.

Ans When source moves with respect to observer in such a way that distance between source and observer does not change, then frequency will not change.

Examples:

I. When source moves in a circle and observer is at rest at the centre.

II. When observer moves in a circle and source is at rest at the centre.

7.5 Why is it difficult to recognize beats when the frequency difference is greater than 10 Hz. Exemplify.

Ans Beats occur due to a slight difference in frequencies causing periodic variations in sound intensity. It is observable when difference of frequencies is less than 10 Hz. When difference becomes greater than 10 Hz then beats are too fast to distinguish, and it becomes difficult for human ear to distinguish it.

Example: If two frequencies with a difference of 20 Hz are sounded together, it would be very hard to even to recognize the beats.

Comprehensive Questions

7.1 State and explain the principle of superposition of waves. Apply this principle to elaborate the working of noise cancelling headphones.

Ans Statement: If a particle of the medium is simultaneously acted upon by two waves, then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called principle of superposition.

Explanation:

(i) Suppose two waves having same phase are travelling in the same direction, with amplitudes y_1 and y_2 , as shown in figure. When these waves overlap each other, their resultant displacement will be:

$$y = y_1 + y_2$$

If $y_1 = y_2$, then resultant displacement will be:
 $y = 2y_1$, or $y = 2y_2$

(ii) If two waves having opposite phase cross each other, their resultant displacement will be:

$$y = y_1 + (-y_2)$$

$$y = y_1 - y_2$$

If $y_1 = y_2$, then resultant displacement will be
 $y = 0$

Thus, if a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves be $y_1, y_2, \dots, y_n$, then the resultant displacement y of the particle is the algebraic sum of all the displacements, i.e.,

$$y = y_1 + y_2 + \dots + y_n$$

This is the principle of superposition of waves.

Mathematically, this can also be written as,

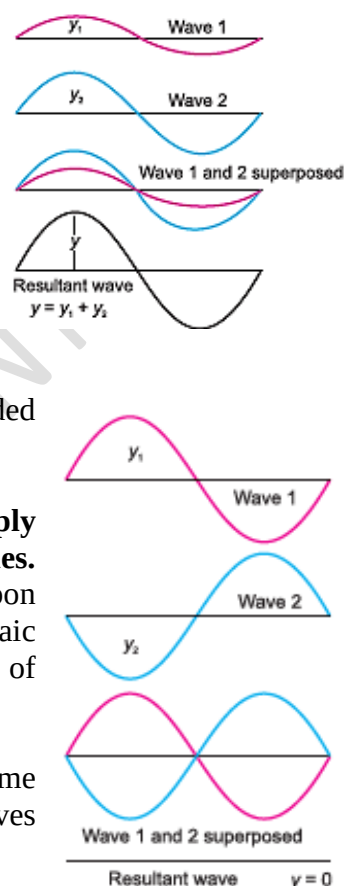
$$y(x, t) = y_1(x, t) + y_2(x, t) + \dots + y_n(x, t)$$

where $y(x, t)$ is the resultant wave and $y_1(x, t), y_2(x, t), \dots, y_n(x, t)$ are the individual waves.

In the context of waves, $y(x, t)$ represents the wave function or wave displacement at a given point x and time t . It describes the shape and evolution of wave over time.

The principle of superposition applies to linear waves or small amplitude waves.

Applications of the Principle of Superposition



By applying the principle of superposition of waves, noise-cancelling headphones effectively eliminate unwanted noise, providing a more immersive and peaceful listening experience.

1. The headphones contain one or more microphones that capture surrounding noise (like background chatter or any environmental noise).
2. The microphone sends the sound signals to an amplifier and a processing unit in the headphones.
3. The processing unit generates an "anti-noise" signal, which is the exact opposite of the surrounding noise (in terms of amplitude and phase).
4. The anti-noise signal is then played through the headphones, along with the desired audio (like music or voice).
5. When the anti-noise signal meets the surrounding noise, the two waves cancel each other out resulting in a much quieter listening experience.

As noise-cancelling headphones use complex algorithms and multiple microphones to achieve optimal noise cancellation, the basic principle of superposition is a fundamental concept in understanding how they work.

7.2 What are standing waves? Illustrate a detailed experiment that demonstrates the standing waves using stretched strings.

Ans Standing waves or Stationary waves:

When two waves of same frequency and amplitude, travelling in opposite directions superpose each other, the resultant wave is called stationary wave, also known as standing waves.

Examples: waves on a string, and sound waves in a pipe.

The term "standing wave" describes that the wave pattern remains fixed in space, without moving forward or backward.

Standing (Stationary) waves on a stretched string

Explanation:

Consider a string of length ℓ stretched between two clamps so that tension in the string is F . When we pluck the string, it vibrates in different loops giving rise to stationary waves. The speed v of the waves produced in the string is given by

$$v = \sqrt{\frac{F}{m}} \quad \text{--- (1)}$$

Here mass per unit length of the string is represented by m .

(a) String Plucked at its Middle Point

Suppose we pluck the string at its middle point, two transverse waves will originate from this point. One moves towards right end other moves towards left end of the string. They are reflected back from the clamped ends and superimpose each other giving rise to stationary waves. Since no motion will take place at the clamped ends, nodes will be formed at the fixed ends of the string. In this case, the string vibrates in a single loop as shown in the figure. We can see that

$$l = \frac{\lambda_1}{2}$$

$$\text{or } \lambda_1 = 2l \quad \text{--- (2)}$$

$$\text{We know that } v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

Putting the value of λ_1 ,

$$f_1 = \frac{v}{2l} \quad \text{--- (3)}$$

Putting the value of v from eq. (1) in eq. (3)

$$f_1 = \frac{1}{2l} \sqrt{\frac{F}{m}}$$

Thus in the first mode of vibration, only fundamental frequency f_1 will be setup.

(b) String Plucked at Quarter Length

When the string is plucked from one quarter of its length it vibrates in two loops as shown in figure 2.

$$\text{Here } l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

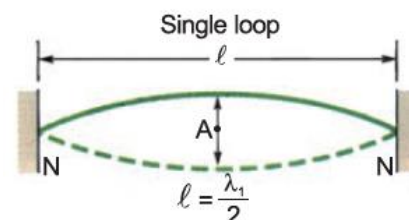


Fig. 1

$$\lambda_2 = \frac{2l}{2} \text{ --- (4)}$$

$$\lambda_2 = l \text{ --- (5)}$$

As

$$v = f_2 \lambda_2$$

$$f_2 = \frac{v}{\lambda_2} \text{ --- (6)}$$

Putting value of λ_2 from eq. (5) in eq. (4),

$$f_2 = \frac{2v}{2l} \text{ --- (7)}$$

$$f_2 = 2 \left(\frac{v}{2l} \right)$$

$$f_2 = 2 f_1 \quad (\because f_1 = \frac{v}{2l})$$

This is known as the second harmonic or first overtone.

(c) String Plucked at an Arbitrary Point

When the string vibrates in n number of loops with $(n+1)$ nodes and n antinodes, then

$$\lambda_n = \frac{2l}{n} \quad \text{where } n = 1, 2, 3, 4, \dots$$

The frequency on the string will be

$$f_n = n \left(\frac{v}{2l} \right)$$

$$f_n = n f_1 \quad (\because f_1 = \frac{v}{2l})$$

Conclusion:

As the string vibrates in more than one loop, its frequency f goes on increasing and the wavelength λ gets correspondingly shorter. However, the product of frequency f and wavelength λ is always equal to v , the speed of waves.

The above discussion clearly establishes that:

1. The stationary waves have a discrete set of frequencies $f_1, 2f_1, 3f_1, \dots, nf_1$ which is known as **Harmonic series**. The lowest characteristic frequency, the fundamental frequency f_1 , corresponds to the first harmonic. The frequency $f_2 = 2f_1$ corresponds to the second harmonic and so on.

2. Thus, quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the **Quantization of frequencies**. It means $f_n = n f_1$ where $n = 1, 2, 3, \dots$ (Integral multiples). The stationary waves can be set up on the string only with the frequencies of harmonic series. Waves which are not in harmonic series are quickly damped out.

3. The frequency of a string on a **musical instrument** can be changed either by varying the tension or by changing the length. For example, the tension in guitar and violin strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck.

7.3 Find the frequencies of the harmonics produced in an organ pipe when it is open at both ends and when it is closed at one end.

Ans Organ Pipe: An organ pipe is a wind instrument in which sound is produced due to setting up of stationary waves in air column. It consists of a hollow long tube with both ends open or with one end open and the other closed.

Modes of Vibrations

Stationary longitudinal waves occur in a pipe as discussed by the following two cases:

Case (1): Modes of Vibrations in an organ pipe open at both ends

Consider an organ pipe of length ℓ open at both ends. The air molecules have complete freedom of motion at the open end, so it acts as an antinode as shown in figure.

(a) Fundamental Mode of Vibration:

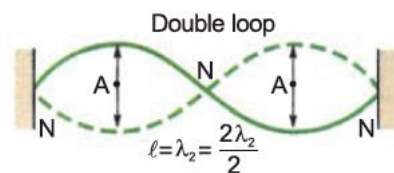


Fig. 2

In this case, there is only one node N at the middle of pipe. As both ends are open, there are two antinodes. Let f_1 be the frequency and λ_1 be the wavelength of sound in the first mode of vibration. From the figure (a),

$$\begin{aligned} \ell &= \frac{\lambda_1}{4} + \frac{\lambda_1}{4} \\ \ell &= \frac{\lambda_1 + \lambda_1}{4} = \frac{2\lambda_1}{4} = \frac{\lambda_1}{2} \\ \lambda_1 &= 2\ell \text{ ----- (1)} \\ \text{As } v &= f_1 \lambda_1 \\ f_1 &= \frac{v}{\lambda_1} \end{aligned}$$

Putting value of λ_1 ,

$$f_1 = \frac{v}{2\ell} \text{ ----- (2)}$$

This is called fundamental frequency or first harmonic.

(b) Second Mode of Vibration:

As shown in figure (b), there are three antinodes and two nodes. So, the frequency will be twice the fundamental frequency in the second mode of vibration.

From the figure (b)

$$\begin{aligned} \ell &= \frac{\lambda_2}{4} + \frac{\lambda_2}{4} + \frac{\lambda_2}{4} + \frac{\lambda_2}{4} = \frac{4\lambda_2}{4} = \frac{2\lambda_2}{2} \\ \lambda_2 &= \frac{2\ell}{2} \text{ ----- (3)} \\ \text{As } v &= f_2 \lambda_2 \\ f_2 &= \frac{v}{\lambda_2} \\ \text{or } f_2 &= 2 \left(\frac{v}{2\ell} \right) \\ f_2 &= 2 f_1 \quad (\because f_1 = \frac{v}{2\ell}) \end{aligned}$$

This is called second harmonics.

(c) nth Mode of Vibration:

The wavelength of air column vibrating in n loops is

$$\lambda_n = \frac{2\ell}{n} \quad \text{where } n = 1, 2, 3, \dots$$

Similarly, the frequency is,

$$f_n = n \left(\frac{v}{2\ell} \right) \quad \text{where } n = 1, 2, 3, \dots$$

Or

$$f_n = n f_1$$

So, the longitudinal stationary waves have a discrete set of frequencies $f_1, 2f_1, 3f_1, \dots, nf_1$, which is known as harmonic series.

Case (2): Modes of Vibration in an Organ Pipe closed at one end

Consider an organ pipe of length ℓ closed at one end. We get node at the closed end and antinode at the open end.

(a) Fundamental Mode of Vibration:

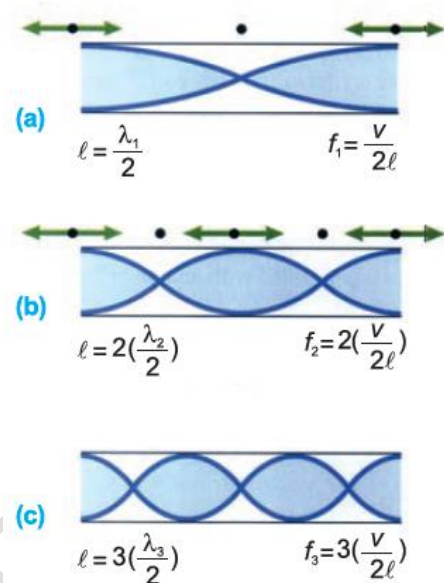
There is one node and one antinode in the fundamental mode of vibration. Let f_1 be the frequency and λ_1 be the wavelength in the first mode of vibration, then from the figure (a)

$$\begin{aligned} \ell &= \frac{\lambda_1}{4} \\ \lambda_1 &= 4\ell \text{ ----- (1)} \end{aligned}$$

$$\begin{aligned} \text{As } v &= f_1 \lambda_1 \\ f_1 &= \frac{v}{\lambda_1} \end{aligned}$$

Putting the value of λ_1 ,

$$f_1 = \frac{v}{4\ell} \text{ ----- (2)}$$



This frequency f_1 is called fundamental frequency.

(b) Second Mode of Vibration:

Second mode of vibration contains two nodes and two antinodes. If λ_3 is the wavelength, then from figure (b)

$$\ell = \frac{\lambda_3}{4} + \frac{\lambda_3}{4} + \frac{\lambda_3}{4}$$

$$\ell = \frac{3\lambda_3}{4}$$

$$\lambda_3 = \frac{4\ell}{3} \text{ -----(3)}$$

As

$$v = f_3 \lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

or

$$f_3 = 3 \left(\frac{v}{4\ell} \right) \text{ ----- (4)}$$

$$f_3 = 3 f_1$$

This is called second harmonic.

(c) nth Mode of Vibration:

If air column vibrates in n loops, the wavelength is,

$$\lambda_n = \frac{4\ell}{n}$$

where $n = 1, 3, 5, 7, \dots$

and frequency is,

$$f_n = n \left(\frac{v}{4\ell} \right) \text{ where } n = 1, 3, 5, 7, \dots$$

or

$$f_n = n f_1$$

From both cases, we conclude that the pipe open at both ends is richer in harmonics.

7.4 Define and exemplify diffraction of waves. Describe this phenomenon by ripple tank experiment.

Ans Diffraction of Waves: It is the bending of waves around the sharp edges or corners of obstacles OR the spreading of waves beyond a barrier.

Condition:

It occurs when a wave strikes a barrier or an opening (a slit) that is comparable in size to the wavelength of the wave.

Examples:

It can be observed in water waves, sound waves, light waves, and electromagnetic waves etc.

Some examples of the phenomenon of diffraction include:

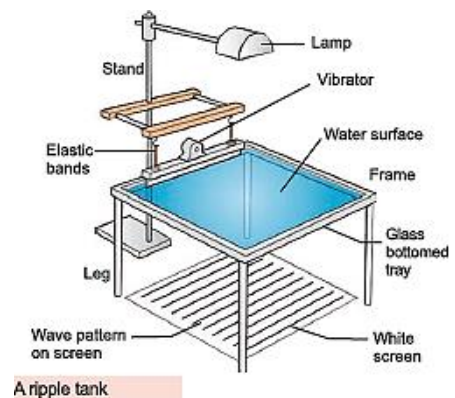
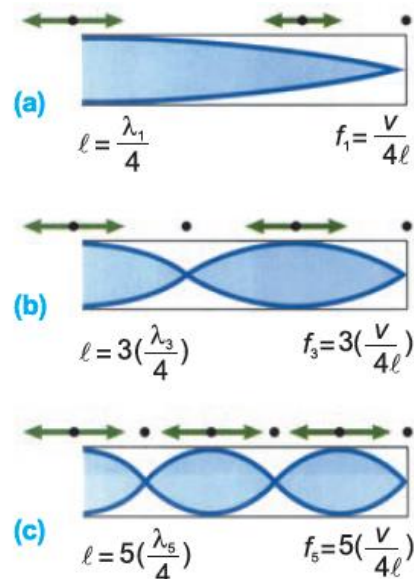
- Hearing sound around a corner.
- Diffraction of X-rays by crystals.
- Water waves bending around obstacles in a ripple tank.

The Ripple Tank

Ripple tank is a very useful apparatus to generate water waves, and to demonstrate wave properties such as reflection, diffraction and refraction.

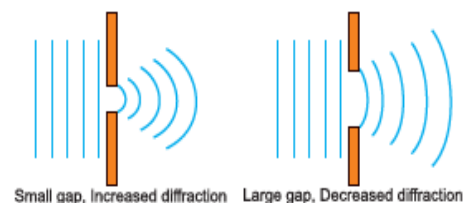
Construction and Working:

Ripple tank contains i) water, ii) vibrator (a motorized oscillating needle), iii) obstacles (a small rectangular block or a semicircular barrier) and iv) gap widths of different sizes.



It creates a series of concentric circles or parallel waves using the vibrator. An obstacle is placed for creating a gap with a specific width. The experiment can be repeated with different gap widths.

It is observed that when the gap width is small compared to the wavelength, diffraction is significant. The waves bend around the obstacle, creating a semicircular pattern. As the gap width increases compared to wavelength, diffraction decreases. The waves pass through the gap with less bending.



Conclusion:

This experiment demonstrates the qualitative effect of gap width on diffraction and wave behavior in a ripple tank.

7.5 What is meant by the term beats? Prove that number of beats per second is equal to the difference between the frequencies of vibrating tuning forks.

Ans Beats: When two waves of slightly different frequencies, travelling in the same direction overlap each other then there is a periodic vibration of sound between maximum and minimum loudness which is called beats.

Explanation:

Consider two tuning forks A and B having the same frequency say 32 Hz are sounded separately, they will give out pure notes. If they are sounded simultaneously, it will be difficult to differentiate the notes of the two individual tuning forks. If the prongs of tuning fork B are loaded with some wax or plasticine, its frequency will be lowered slightly, say it becomes 30 Hz. Now when these two tuning forks are sounded together, a note of increasing and decreasing intensity will be heard. This is called a beat which is due to interference of sound waves from the two tuning forks.

Figure (a) and (b) show the waveforms of the notes emitted from tuning forks A and B respectively. When both tuning forks are sounded together, the resultant waveform is shown in figure (c).

At some instant X, the displacement of the two waves is in the same direction. At this point, the displacement is large, and a loud sound is heard.

After $\frac{1}{4}$ s the displacements of the two waves are opposite to each other, resulting in minimum displacement at Y. Hence faint or no sound is heard.

Another $\frac{1}{4}$ s later, the displacements are again in the same direction, and a loud sound is heard again at point Z.

Thus, during one second, two faintest or two loudest sounds are observed. As the difference of frequencies is 4 Hz, we find that the number of beats per second is equal to the difference in frequencies of the two tuning forks.

$$\begin{aligned}\text{No. of beats} &= f_A - f_B \\ n &= (32 - 30) \text{ Hz} = 2\end{aligned}$$

Condition:

When the difference between frequencies is greater than 10 Hz, it becomes difficult to recognize the beats.

7.6 What do you understand by progressive waves? Discuss the intensity of progressive waves.

Ans Progressive Waves: A progressive wave or travelling wave is a wave that travels through a medium in a consistent direction and transfers energy from one point to another.

Examples:

Water waves, sound waves, light waves, etc are progressive waves.

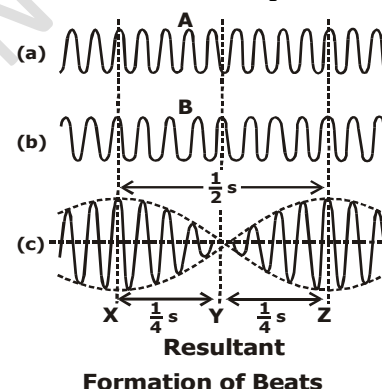
Intensity of Wave:

It is the amount of energy transmitted per unit area per unit time in the direction of propagation of progressive wave.

It is a measure of the power of a wave and is denoted by "I".

Formula:

The intensity of a wave is given by:



$$I = \frac{E}{A \times t} = \frac{E/t}{A}$$

or

$$I = \frac{P}{A} \quad (\because \frac{E}{t} = P)$$

Here E = Energy in joules (J), t = Time in seconds (s), P = Power in watts (W)

Unit:

Intensity is measured in watts per square metre (W m^{-2}).

Explanation:

We know that in mechanical waves (e.g. sound waves, water waves, waves on a vibrating string), energy is stored as kinetic and potential energy of the medium's particles. The amount of energy stored depends upon the displacement (amplitude) of the particles from the mean position.

Therefore, the intensity I of waves is proportional to the square of the amplitude A i.e.,

$$I \propto A^2$$

$$I = k A^2$$

Here k is the constant of proportionality. It depends upon the physical properties of the wave and the medium.

7.7 Keeping in mind “Doppler effect”, analyze the following cases:

(a) when source of sound moves away from the stationary observer.

(b) when source of sound moves towards the stationary observer.

Ans The apparent change in the frequency (or pitch) of waves due to the relative motion between the source and observer is called Doppler Effect.

Explanation:

Consider a source of sound S at rest which emits sound waves having wavelength λ . Let speed of sound for a stationary observer is v, the frequency is given by

$$v = f \lambda$$

or

$$f = \frac{v}{\lambda} \text{----- (1)}$$

$$\lambda = \frac{v}{f} \text{----- (2)}$$

Case 1: When source of sound moves towards the stationary observer

When a source moves towards stationary observer A with a velocity u_s , then sound waves are compressed, and their wavelength is decreased as shown in figure. The waves are compressed by an amount given as

$$\Delta \lambda = \left(\frac{u_s}{f} \right)$$

where $\Delta \lambda$ is called Doppler Shift.

The compression of waves is due to the fact that same numbers of waves are occupied in shorter space depending upon velocity of the source. The wavelength observed by the observer A is given by

$$\lambda_A = \lambda - \Delta \lambda$$

$$= \frac{v}{f} - \frac{u_s}{f}$$

$$\lambda_A = \frac{v - u_s}{f}$$

Thus, the apparent frequency in this case is given by

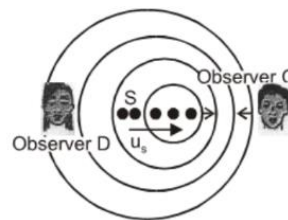
$$f_A = \frac{v}{\lambda_A}$$

Putting value of λ_A ,

$$f_A = \frac{v}{\frac{v - u_s}{f}}$$

$$f_A = \left[\frac{v}{v - u_s} \right] f \text{----- (5)}$$

As $\left[\frac{v}{v - u_s} \right] > 1$, So $f_A > f$



Result: Thus, the apparent frequency of sound is increased when the source moves towards the observer.

Case 2: When source of sound moves away from the stationary observer

When the source moves away from the stationary observer B with a velocity u_s , then sound waves are expanded, and their wavelength is increased. The waves expand by an amount given as

$$\lambda_B = \left(\frac{u_s}{f} \right)$$

The expansion of waves is due to the fact that the same numbers of waves are occupied in a greater space. This happens due to the velocity of the source. The new wavelength of the waves is given by,

$$\begin{aligned}\lambda_B &= \lambda + \Delta\lambda \\ &= \frac{v}{f} + \frac{u_s}{f} \\ \lambda_B &= \frac{v+u_s}{f}\end{aligned}$$

The apparent frequency in this case is given by

$$\begin{aligned}f_B &= \frac{v}{\lambda_B} \\ f_B &= \left[\frac{v}{(v+u_s)/f} \right] \\ f_B &= \left[\frac{v}{(v+u_s)} \right] f \text{-----(6)}\end{aligned}$$

As $\frac{v}{v+u_s} < 1$, so $f_B < f$

Result: Therefore, apparent frequency of sound is decreased when source moves away from observer.

Numerical Problems

7.1 The speed of a wave on a typical string is 24 m s^{-1} . What driving frequency will it resonate if its length is 6.0 m ?

Given Data:

$$v = 24 \text{ m s}^{-1}$$

$$l = 6.0 \text{ m}$$

To Find:

$$f = ?$$

Solution: Wavelength for fundamental frequency is:

$$\begin{aligned}\lambda &= 2l \\ &= 2 \times 6.0 = 12.0 \text{ m}\end{aligned}$$

Using wave equation,

$$f = \frac{v}{\lambda} = \frac{24}{12}$$

$$f = 2 \text{ Hz}$$

Ans

7.2 The lowest resonance frequency for a guitar string of length 0.75 m is 400 Hz . Calculate the speed of a transverse wave on the string.

Given Data:

$$l = 0.75 \text{ m}$$

$$f = 400 \text{ Hz}$$

To Find:

$$v = ?$$

Solution:

Wavelength in fundamental mode

$$\lambda = 2l$$

$$= 2 \times 0.75$$

$$\lambda = 1.5 \text{ m}$$

Using wave equation,

$$v = f \times \lambda$$

$$= 400 \times 1.5$$

$$v = 600 \text{ m s}^{-1} \quad \text{Ans}$$

7.3 A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

Given Data:

$$f_A = 320 \text{ Hz}$$

$$\text{no. of beats after loading} = 6$$

To Find:

$$\text{Frequency of B after loading} = f_B = ?$$

Solution: We know that

$$f_A - f_B = n \quad \text{or} \quad f_B = f_A - n$$

After loading, the beat frequency becomes 6 beats per second. Therefore,

$$f_B = f_A - n$$

$$f_B = 320 - 6$$

$$f_B = 314 \text{ Hz} \quad \text{Ans}$$

7.4 A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked. Density of steel wire is $7.8 \times 10^3 \text{ kg m}^{-3}$.

Given Data:

$$F = 80 \text{ N}$$

$$l = 1.5 \text{ m}$$

$$d = 0.50 \text{ mm} = 0.0005 \text{ m}$$

$$\rho = 7.8 \times 10^3 \text{ kg m}^{-3}$$

To Find:

$$\text{Fundamental frequency} = f = ?$$

Solution:

$$\text{Area} \quad A = \frac{\pi d^2}{4}$$

$$= \frac{(3.14)(0.0005)^2}{4} = 1.9635 \times 10^{-7} \text{ m}^2$$

$$\text{Mass per unit length } m = \rho \times A$$

$$= 7.8 \times 10^3 \times 1.9635 \times 10^{-7}$$

$$= 0.00153 \text{ kg m}^{-1}$$

$$\text{Wave speed} \quad v = \sqrt{\frac{F}{m}}$$

$$= \sqrt{\frac{80}{0.00153}} = 228.4 \text{ m s}^{-1}$$

Fundamental frequency

$$f = \frac{v}{2l}$$

$$= \frac{228.4}{2 \times 1.5}$$

$$f = 76.2 \text{ Hz} \quad \text{Ans}$$

7.5 Average intensity of sunlight on the surface of the Earth is nearly 500 W m^{-2} . Determine the amount of energy that falls on a solar panel having an area of 0.50 m^2 in four hours.

Given Data:

$$I = 500 \text{ W m}^{-2}$$

$$A = 0.50 \text{ m}^2$$

$$t = 4 \text{ hours} = 4 \times 3600 = 14400 \text{ s}$$

To Find:

Energy received = $E = ?$

Solution:

$$\begin{aligned}\text{Energy } E &= I \times A \times t \\ &= 500 \times 0.5 \times 14400\end{aligned}$$

$$E = 3.6 \times 10^6 \text{ J} \quad \text{Ans}$$

7.6 (a) If the intensity of a wave is 16 W m^{-2} and the amplitude is 2 m, what is the value of constant k ?

(b) If the intensity of a wave is 25 W m^{-2} and the constant k is 5 W m^{-4} , what is the amplitude?

(a) Given Data:

$$\text{Intensity } I = 16 \text{ W m}^{-2}$$

$$\text{Amplitude} = A = 2 \text{ m}$$

To Find:

Constant $k = ?$

Solution:

$$I = k A^2 \Rightarrow k = \frac{I}{A^2}$$

$$k = \frac{16}{2^2}$$

$$k = 4 \text{ W m}^{-4} \quad \text{Ans}$$

(b) Given Data:

$$\text{Intensity } I = 25 \text{ W m}^{-2}$$

$$\text{Constant } k = 5 \text{ W m}^{-4}$$

To Find:

$$\text{Amplitude } A = ?$$

Solution:

$$I = k A^2 \Rightarrow A^2 = \frac{I}{k}$$

$$= \frac{25}{5} = 5$$

$$\Rightarrow \sqrt{A^2} = \sqrt{5}$$

$$A = 2.24 \text{ m} \quad \text{Ans}$$

7.7 (a) A sound system produces 200 watts of power. If the sound is directed at a crowd with an area of 150 m^2 , what is the intensity of sound?

(b) A light bulb emits 100 watts of power. If the light is spread out evenly over a sphere with a surface area of 400 m^2 , what is the intensity of light?

(a) Given Data:

$$P = 200 \text{ W}$$

$$A = 150 \text{ m}^2$$

To Find:

$$\text{Intensity of sound} = I = ?$$

Solution:

$$\begin{aligned}\text{Intensity } I &= \frac{\text{Power}}{\text{Area}} \\ &= \frac{200}{150} = 1.33\end{aligned}$$

$$I = 1.33 \text{ W m}^{-2} \quad \text{Ans}$$

(b) Given Data:

$$\text{Power} = P = 100 \text{ W}$$

$$\text{Area} = A = 400 \text{ m}^2$$

To Find:

$$\text{Intensity of light} = I = ?$$

Solution:

$$\begin{aligned}\text{Intensity } I &= \frac{\text{Power}}{\text{Area}} \\ &= \frac{100}{400}\end{aligned}$$

$$I = 0.25 \text{ W m}^{-2} \quad \text{Ans}$$

7.8 A radio antenna broadcasts 500 watts of power. If the signal is received at a distance of 10 km, what is the intensity of the signal?

Given Data:

$$P = 500 \text{ W}$$

$$r = 10 \text{ km} = 10,000 \text{ m}$$

To Find:

$$I = ?$$

Solution: As signal spreads spherically,

$$\text{Area} = 4 \pi r^2$$

$$= 4 \pi (10000)^2$$

$$A = 1.26 \times 10^9 \text{ m}^2$$

$$\begin{aligned} \text{Intensity} = I &= \frac{\text{Power}}{\text{Area}} \\ &= \frac{500}{(1.26 \times 10^9)} \end{aligned}$$

$$I = 4 \times 10^{-7} \text{ W m}^{-2} \quad \text{Ans}$$

7.9 An organ pipe has a length of 1 m. Determine the frequencies of the fundamental and the first two harmonics:

(a) if the pipe is open at both ends

(b) if the pipe is closed at one ends.

(Speed of sound in air is 340 ms^{-1})

Given Data:

$$l = 1 \text{ m}$$

$$v = 340 \text{ m s}^{-1}$$

To Find:

$$f_1 = ?$$

$$f_2 = ?$$

$$f_3 = ?$$

Solution:

(a) For pipe open at both ends:

$$f_1 = \frac{v}{2l}$$

$$f_1 = \frac{340}{2} = 170 \text{ Hz}$$

$$\text{and } f_2 = 2 f_1 = 340 \text{ Hz}$$

$$f_3 = 3 f_1 = 510 \text{ Hz}$$

(b) For pipe closed at one end:

$$f_1 = \frac{v}{4l}$$

$$f_1 = \frac{340}{4} = 85 \text{ Hz}$$

$$\text{and } f_3 = 3 f_1 = 255 \text{ Hz}$$

$$f_5 = 5 f_1 = 425 \text{ Hz}$$

7.10 A train is approaching a station at 90 km h^{-1} , sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed? (Speed of sound is 340 ms^{-1})

Given Data:

$$\text{Speed of train} = u_s = 90 \text{ km h}^{-1} = \frac{90 \times 1000}{3600} = 25 \text{ ms}^{-1}$$

$$\text{Frequency} = f = 1000 \text{ Hz}$$

$$\text{Speed of sound } v = 340 \text{ ms}^{-1}$$

To Find:

$$\text{Apparent frequency for approaching} = f_{\text{approach}} = ?$$

Apparent frequency for receding = $f_{\text{receding}} = ?$

Solution:

(a)
$$f_{\text{approach}} = f \times \frac{v}{(v - u_s)}$$
$$= 1000 \times \frac{340}{340 - 25}$$
$$f_{\text{approach}} = 1079.4 \text{ Hz} \quad \text{Ans}$$

(b)
$$f_{\text{receding}} = f \times \frac{v}{(v + u_s)}$$
$$= 1000 \times \frac{340}{340 + 25}$$
$$f_{\text{receding}} = 931.5 \text{ Hz} \quad \text{Ans}$$

PHYSICS DEPARTMENT FCC

Chapter No 8 Physical Optics and Gravitational Waves

Exercise

MCQs

8.1 The phenomenon of polarization of light is:

- (a) the process of scattering of light (b) the property of light to vibrate in a specific plane
(c) the ability of light to travel in a straight line (d) the phenomenon of light changing color

8.2 Malus's law states that:

- (a) the intensity of light is directly proportional to the square of the cosine of the angle between the light wave and the analyzer
(b) the intensity of light is directly proportional to the square of the sine of the angle between the light wave and the analyzer
(c) the intensity of light is directly proportional to the angle between the light wave and the analyzer
(d) the intensity of light is inversely proportional to the angle between the light wave and the analyzer

8.3 The intensity of light when it passes through a polarizer:

- (a) increases (b) decreases (c) remains the same (d) becomes zero

8.4 The angle between the light wave and the analyzer is called:

- (a) polarization angle (b) refraction angle (c) reflection angle (d) azimuth angle

8.5 The key purpose of an analyzer in a polarization experiment is:

- (a) to polarize the light (b) to measure the intensity of light
(c) to change the direction of light (d) to filter out unwanted light

8.6 The mathematical representation of Malus's law is:

- (a) $I = I_0 \cos^2 \theta$ (b) $I = I_0 \sin^2 \theta$ (c) $I = I_0 \tan^2 \theta$ (d) $I = I_0 \cot^2 \theta$

8.7 The effect of increasing the angle between the light wave and the analyzer on the intensity of light is:

- (a) the intensity increases (b) the intensity decreases
(c) the intensity remains the same (d) the intensity becomes zero

8.8 The condition for maximum intensity of light in a polarization experiment is when:

- (a) the light wave and analyzer are perpendicular (b) the light wave and analyzer are parallel
(c) the light wave and analyzer are at an angle of 45°
(d) the light wave and analyzer are at an angle of 60°

8.9 The unwanted light that interferes with vision is termed as:

- (a) haze (b) glare (c) contrast (d) flare

8.10 Who predicted the existence of gravitational waves?

- (a) Galileo Galilei (b) Albert Einstein (c) Issac Newton (d) Leonardo da Vinci

8.11 What are gravitational waves?

- (a) Electromagnetic waves (b) Mechanical waves
(c) Ocean waves (d) Ripples in the fabric of space time

8.12 Which is the primary method used to detect gravitational waves?

- (a) Optical telescopes (b) Radio telescopes (c) LASER interferometry (d) Gravitational lensing

8.13 Which of the following is a primary source of gravitational waves?

- (a) Binary black hole merger (b) Solar flares (c) Earthquake (d) Solar wind

Answer Key

Q.No.	Option	Q.No.	Option	Q.No.	Option	Q.No.	Option
8.1	B	8.5	B	8.9	B	8.13	A
8.2	A	8.6	A	8.10	B		

8.3	B	8.7	B	8.11	D		
8.4	A	8.8	B	8.12	C		

Short Answer Questions

8.1 Why are polaroid sunglasses better than ordinary sunglasses?

Ans Polaroid sunglasses:

1. Pass light having specific planes of vibration only.
2. Reduce the glare.
3. Reduce the intensity of light with less effect on vision.
4. Save eyes from unnecessary strain.

8.2 Is light from the sky partially polarized? How is it so?

Ans Yes, light from the sky is partially polarized.

When sunlight interacts with the air molecules and other particles of atmosphere, it scatters in different directions. This scattering causes the polarization of light. Polarization is maximum at an angle of 90° .

8.3 How is Malus's law used in everyday life?

Ans Malus's law has several practical applications in everyday life:

- **Polarized sunglasses:** They reduce glare from reflective surfaces like water, snow and car windshields.
- **Camera Filters:** Polarizing filters are used in photography to reduce glare and enhance colors.
- **LCD screens:** LCD screens in smart phones, tablets, and TVs, polarize the light.
- **Biological Systems:** It is used to study the polarization of light in biological systems such as cell membranes.

8.4 What are the applications of Brewster's angle?

Ans Brewster's angle is used to reduce the glare and improve the image quality in:

1. polarizing filters in photography.
2. lasers to optimize design and minimize reflection losses.
3. microscopes to enhance contrast.
4. coatings for car windshields and sunglasses.

8.5 What is the space-time curvature?

Ans Space time curvature is the bending of space and time around massive objects, such as planets and stars. This is caused by the gravitational pull of massive objects. The curvature of space time affects the motion of objects and light, causing them to follow curved paths. It is a fundamental concept in Einstein's theory of general relativity.

Constructed Response Questions

8.1 Write down some applications of plane polarized light.

Ans Some applications of plane polarized light are:

1. **Sunglasses:** Plane polarized sunglasses reduce the glare and intensity of light.
2. **Optical instruments:** Polarizers are used in cameras and microscopes to enhance contrasts and reduce reflections.
3. **LCD Screens:** Polarizers control the orientation of liquid crystals to create images on the screen.

8.2 Would it be possible to use a polarizer as an analyzer? If yes, give at least two examples.

Ans Yes, a polarizer can be used as an analyzer. An Analyzer is also a polarizer that is placed after the polarizer. Rotation of analyzer controls the intensity of the polarized light, by adjusting the angle of analyzer with respect to the polarizer.

Examples: 1. Tourmaline 2. Calcite crystals

8.3 Explain how Malus's law is used in the design of polarized sunglasses. How do these surfaces reduce glare from reflective surface? Provide an example to illustrate your answer.

Ans Polarized sunglasses are used to reduce glare from reflective surfaces using the Malus's law. According to this law, the intensity of light transmitted through a polarizer is given by

$$I = I_0 \cos^2$$

In polarized sunglasses there are polarizing filters. When filters are parallel to the plane of polarized light, the intensity of transmitted light remains same i.e., $\theta = 0^\circ$, $I = I_0$.

As the polarizers are rotated, intensity starts decreasing. When polarizers are perpendicular to the plane of polarized light, the light vanishes i.e., $\theta = 90^\circ$, $I = 0$.

Example: On a bright sunny day, the road's reflection can cause intense glare which can be reduced by polaroid sunglasses.

8.4 How will the sky appear if there had been no atmosphere?

Ans If there were no atmosphere above earth the sky would appear black, even during the day. The sky appears blue due to scattering of sunlight from the molecules in the atmosphere.

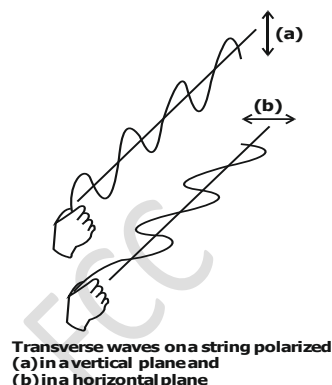
8.5 What is the significance of detecting gravitational waves?

Ans Gravitational waves provide comprehensive understanding of cosmic phenomena.

The gravitational waves are the ripples in space time fabric. They travel at the speed of light and carry the information about the cosmic events like merger of black holes or neutron stars and supernova explosions.

8.6 How are tidal forces formed?

Ans Gravitational waves passing through a body with mass can cause the body to experience periodic stretching and compressing (space time distortion). This effect is known as tidal forces. It is the result of gravitational wave's oscillating nature.



Comprehensive Questions

8.1 Define the phenomenon of polarization of waves? How does polarization of electromagnetic waves occur? Also classify the polarization of waves.

Ans Polarization:

In simple words, the process of transforming an unpolarized light into a polarized light is called polarization.

Physical optics with reference to polarization deals with the behavior of light waves and their interaction with matter.

The phenomenon of interference and diffraction prove that light has wave nature. The phenomenon of polarization shows that light waves are transverse in nature.

In transverse mechanical waves, the vibration of particles of medium are perpendicular to the direction of propagation of waves. The vibration can be oriented along vertical, horizontal or any other direction. In each of these cases the waves are said to be polarized.

The **plane of polarization** is the plane containing the direction of vibration of the particles of the medium and the direction of propagation of a wave.

A light wave produced by oscillating charge consists of a periodic variation of electric field vector along with magnetic field vector at right angle to each other.

Unpolarized Light:

A beam of ordinary light consisting of many planes of vibration is called unpolarized light.

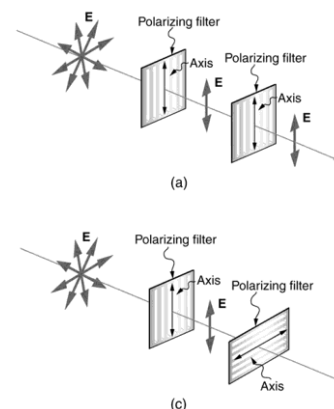
Examples: sunlight, incandescent light bulbs, fluorescent bulbs, light from a candle or fire.

Polarized Light:

The beam of light in which all vibrations are confined to a single plane of vibration is called polarized light.

Polarization is the process by which electric and magnetic vibrations of light waves are restricted to a single plane of vibration. It is the property shown by transverse waves only.

Polarization of Unpolarized Light:



The most common method by which an unpolarized light can be polarized is by passing it through a polarizing filter. Only that electric field vector which is parallel to the axis of polarized filter can pass through it, all other vectors are blocked. The resultant light then becomes polarized as shown in fig.

Types of Polarization:

The direction of the electric field vector of light waves in a specific direction is the basis of polarization. The basic types of polarization of light are:

1. Linear Polarization: When the electric field vector oscillates in a single plane, light is said to be linearly polarized.

Example: Light passing through a polarizing filter like sunglasses.

2. Circular Polarization: When the electric field rotates circularly, either clockwise or anticlockwise, the light is said to be circularly polarized.

Example: Light reflected from a CD or DVD.

3. Elliptical Polarization: When the electric field follows an elliptical path, combining linear and circular polarization, it is called elliptical polarization.

Example: Light passing through a stressplate or waveplate.

8.2 How can the plane polarized light be produced and detected? What does it prove?

Ans Production and detection of plane polarized light:

The light emitted by an ordinary incandescent bulb is unpolarized because its vibrations are randomly oriented in space.

If the unpolarized light is made incident on the sheet of polaroid the transmitted light will be plain polarized.

If a second sheet of polaroid is placed in such a way that the axes of the polaroid as shown by the straight lines drawn on them are parallel the light is transmitted through the second polaroid also. If the second polaroid is slowly rotated about the beam of light as axis of rotation, the light emerging from the second polaroid gets dimmer and dimmer and disappears when the axes become mutually perpendicular.

The light re-appears on further rotation and becomes brightest when the axes are again parallel to each other.

This experiment proves that light waves are transverse waves. If the light waves are longitudinal, they would never disappear even if the two polaroids were mutually perpendicular.

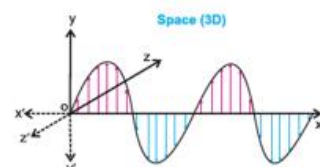
8.3 How can polarized light be obtained by the method of reflection. Explain.

Ans In 1808, Malus discovered that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. If the reflected light is viewed through a polaroid which is slowly rotated about the line of vision, the light is practically extinguished at a certain orientation of the polaroid.

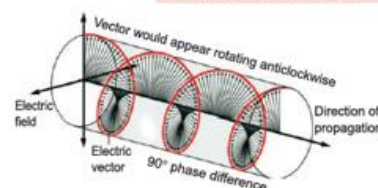
The most suitable angle of incidence 'i' is about 57° for glass for which the reflected ray becomes plane polarized, as illustrated by ray diagram in Figure. This proves that the light reflected by the glass is practically plane polarized.

Light reflected from the surface of a table becomes darker when viewed through a rotated polaroid, showing that it is partially plane polarized.

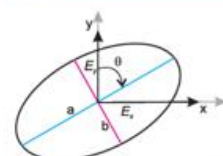
Brewster's Law: The angle of incidence on a transparent medium when the reflected light is almost plane polarized is called the polarizing angle. Let a beam of unpolarized light be made incident on the surface of medium 2 as shown in Fig. If the reflected beam of light is almost plane polarized, the reflected and refracted beams are at right angles to each other at the polarizing angle, $i = \theta_p$. Thus;



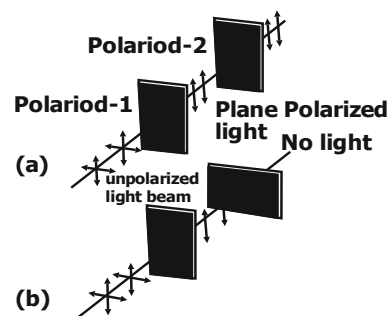
(a): Linear polarization of light



(b): Circular polarization of light



(c): Elliptical polarization of light



Experimental arrangement to show that light waves are transverse. The lines with arrows indicates electric vibrations of light waves.

$$\theta_p + \theta_r = 90^\circ$$

$$\text{or } \theta_r = 90^\circ - \theta_p$$

From Snell's Law,

$$n_1 \sin \theta_p = n_2 \sin \theta_r$$

$$n_1 \sin \theta_p = n_2 \sin (90^\circ - \theta_p)$$

$$n_1 \sin \theta_p = n_2 \cos \theta_p$$

$$\frac{\sin \theta_p}{\cos \theta_p} = \frac{n_2}{n_1}$$

$$\tan \theta = \frac{n_2}{n_1}$$

This equation is known as Brewster's law. In this equation, n_1 is refractive index of medium 1 and n_2 is refractive index of medium 2. If medium 1 is air, then equation becomes

$$\tan \theta_p = n$$

because $n_1 = 1$ and $n_2 = n$

Here n is refractive index of medium on which light is incident.

Hence, Brewster proved that the tangent of the angle of polarization is numerically equal to the refractive index of the medium 2 when medium 1 is air.

Brewster's Angle: In Brewster's law, the angle ' θ_p ' for which the reflected ray and the refracted ray make an angle of 90° between them, is also called the Brewster angle θ_B . Then, $\tan \theta_B = n$ holds.

8.4 State Malus's Law. Explain the intensity formula.

Ans Malus's law states that:

The intensity I of plane polarized light after passing through an analyzer is directly proportional to the square of the cosine of the angle θ between the transmission axis of the analyzer and polarizer. That is;

$$I \propto \cos^2 \theta$$

$$I = I_0 \cos^2 \theta$$

where I_0 = Intensity of the incident polarized light.

Malus's law gives a mathematical relation between the intensity of the light incident on the first polaroid (i.e., polarizer) and the intensity of light obtained after passing it through the second polaroid (i.e., analyzer).

Analyzer:

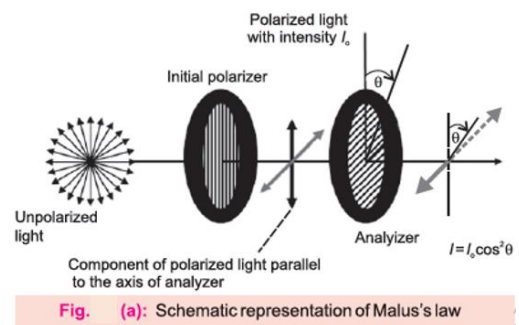
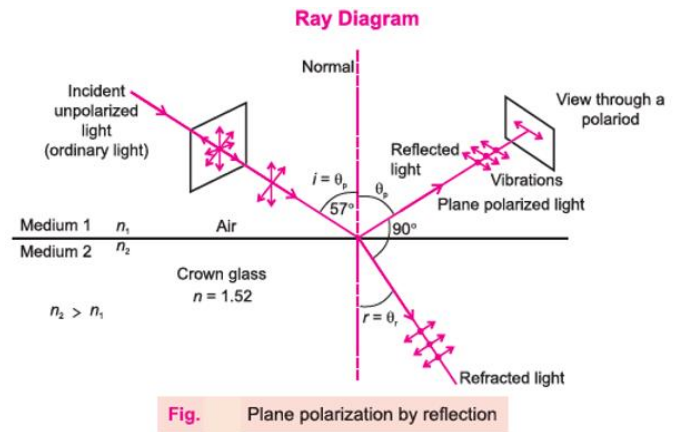
An analyzer is also a polarizer that is placed after a polarizer. Rotation of the analyzer affects the intensity of the polarized light. It is used to further reduce the intensity of light and adjust it by adjusting the angle of the analyzer with respect to the polarizer.

Polaroid:

Certain transparent crystalline materials, like tourmaline, calcite crystals, etc., are capable of confining vibrations of light waves in only one plane. Such materials are called polaroids which have high directionality in crystal structure. Light can also be polarized by natural phenomena like reflection, refraction and scattering.

Intensity of light:

If a piece of polaroid is rotated in front of a polarized ray of light, it causes a variation in the intensity of the light that gets through. The reason that causes the variation of intensity is the angle between the initial polarizer and the axis of second polarizer.



Formula:

When the incident polarized light of amplitude A , strikes the analyzer at an angle θ , it is resolved into two components $A_0 \cos \theta$ and $A_0 \sin \theta$ as shown in Fig. (b). Component $A_0 \sin \theta$ is absorbed in the analyzer. Since, only $A_0 \cos \theta$ passes through the analyzer, the amplitude A of the transmitted light is therefore,

$$A = A_0 \cos \theta$$

Since intensity I is the square of the amplitude A , it can be expressed as:

$$I \propto A^2 \quad \text{or} \quad I = k A^2$$

where $k=1$ So,

$$I = A^2$$

or

$$I = A_0^2 \cos^2 \theta$$

Therefore;

$$I = I_0 \cos^2 \theta \quad (\because I_0 = A_0^2)$$

Here I_0 is the intensity of the incident polarized light.

Conditions:

There are two extreme conditions of θ followed by the above equation given as,

1) If $\theta = 0^\circ$, then $I = I_0$.

This means the intensity transmitted through the analyzer is equal to the initial light intensity that passes through the polarizer.

2) If $\theta = 90^\circ$, then $I = 0$.

This means the light is extinguished completely, i.e., no light is allowed to pass through the analyzer.

8.5 What is a polaroid? Explain two main applications of polarization.

Ans A Polaroid is an optical device that transmits only one component of light vibrations and absorbs the others. They are capable of confining vibrations of light waves in only one plane and thus have high directionality in crystal structure.

Applications of Polarization:

Polarizers, also known as polarizing filters, have two main applications.

1. Sky Photography:

A camera which is used to photograph the clouds is fitted with a polaroid. The light coming from the sky polarized by the polaroid.

In sky photography, polarizers are used to reduce the glare and haze which are produced by the scattering of light by small particles or molecules present in the atmosphere. Polarizers also enhance the contrast by blocking the excessive bright white light while allowing the other colors to pass through, thus creating a brighter detailed image. Thus, it allows us to improve the overall image quality.

2. Stress Analysis of Materials:

In materials science, polarizers are used to analyze the stress and strain on materials, such as plastics, metals, and glass.

When a material is stressed, its molecular structure changes, affecting the way it interacts with light, and interference patterns on fringes are formed which in turn gives qualitative information about the material. By shining polarized light through the transparent material and analyzing the changes in the light's polarization, the

Researchers can:

- determine the material's stress patterns
- identify potential weaknesses or defects
- analyze the material's optical properties
- understand how the material will behave under different conditions. This technique is known as "**photo-elasticity**" and is widely used in fields like engineering, materials science, and quality control.

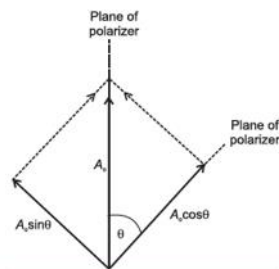
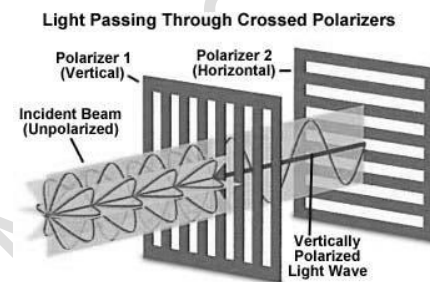


Fig (b): Resolution of amplitude into components of plane polarization light.



In both cases described above, polarizers play a crucial role in manipulating light to achieve specific goals, such as enhancing image quality or analyzing material properties.

8.6 What are gravitational waves? Describe the basic types of gravitational waves.

Ans A gravitational wave is a stretching and compressing of space-time and can be observed by measuring the change in length between two objects.

Gravitational waves (GWs) are:

"Ripples in the fabric of space-time, produced by violent cosmic events, like colliding with black holes or neutron stars that travel at the speed of light, carrying information about their source."

Example:

The simplest example to understand GWs is given below:

If we throw a stone into a pond, the stone creates ripples on the water surface (space-time). These ripples travel outward, carrying information about the stone (the cosmic event). They can be detected on the shore by (gravitational wave observatories), revealing the stone's presence and properties.

Types of Basic Gravitational Waves:

There are four basic types of gravitational waves, each with different sources and characteristics:

1. Continuous Gravitational Waves:

When a single massive object spins at a constant rate, such as a neutron star, continuous GWs are produced with the same frequency and amplitude. White dwarf binary systems produce continuous GWs.

2. Compact Binary Inspirational Gravitational Waves:

When a binary system, such as binary neutron stars, binary black holes, or a neutron star and black hole orbiting each other, compact inspirational GWs are produced.

3. Burst Gravitational Waves:

These are produced by violent events like supernovae, gamma-ray bursts, or cosmic strings.

4. Stochastic Gravitational Waves:

Stochastic GWs are weak, random signals of GWs which are produced by superposition of many weak gravitational wave sources, such as distant binary systems. These GWs are the most difficult to detect. Every physical object that accelerates produces gravitational waves. Vehicles, airplanes, etc. are included in it. The masses and acceleration of objects on the Earth are too small to make gravitational waves big enough to be detected with our instruments.

Binary System:

A system consisting of two compact objects such as black holes, neutron stars, which are orbiting each other and emitting gravitational waves is called a Binary System.

The binary systems are significant sources of gravitational waves, and their mergers are among the most intense cosmic events. As the binary systems orbit each other, they emit gravitational waves, which can be detected by observatories like LIGO (Laser Interferometer Gravitational Wave Observatory). The waves carry information about the system's mass, spin, and merger dynamics, offering insights into these extreme cosmic objects, and move with the speed of light.

The characteristics of GWs depend on the system's properties, such as:

- Masses of the objects
- Orbital period and frequency
- Eccentricity of the orbit: Eccentricity e is a measure of the amount by which an object deviates from a perfect circle.



8.7 What is an interferometer? Describe the basic LIGO interferometer in detail.

Ans GRAVITATIONAL WAVES (GWs) INTERFEROMETER

An interferometer is an optical tool used in detecting gravitational waves. It is a very sensitive detection device that may use the interference of LASER (Light Amplification by Stimulated Emission of Radiation) beams.

LIGO:

An interferometer that detects gravitational waves is a highly sensitive instrument. It uses LASER light to measure tiny changes in distance between mirrors, caused by gravitational waves passing through the

detectors. These interferometers are called LIGO (LASER Interferometer Gravitational Wave Observatory).

Difference:

The main differences between LIGO and conventional interferometers are:

- (a) LIGO is 1000 times larger than conventional devices.
- (b) LIGO uses LASER whereas conventional interferometer has normal light source.

The basic LIGO interferometer can be seen in Figure.

Basic Components of GW Interferometer:

The basic components of a gravitational wave interferometer are:

- 1. **Laser** that produces a stable and high intensity beam of light.
- 2. **Power recycling mirror** continually reflects LASER light that has already travelled through the instrument back into the interferometer and hence the term recycling is used.
- 3. **Beam splitter** divides the LASER beam into two perpendicular beams.
- 4. **Mirrors** reflect the beams, creating two perpendicular arms.
- 5. **Fabry Perot cavity** consists of two mirrors facing each other. The purpose of the cavity is to enhance the path length.
- 6. **Photodetectors** measure the returning beams, detecting tiny phase shifts (if any).
- 7. **Arm cavities** enhance the LASER light, increasing sensitivity.

Working of Interferometer:

A laser beam is split into two perpendicular beams, each travelling down two identical paths (arms) of the interferometer. The beams bounce off mirrors at the ends of each arm and return to the starting point, where they are recombined. If a gravitational wave passes through, it causes a tiny disturbance in the distance between the mirrors, resulting in a phase shift between the two beams.

When the beams recombine, they create an interference pattern, which is measured by a photodetector. The tiny phase shift caused by the gravitational wave alters the interference pattern, allowing the detector to sense the wave's presence.

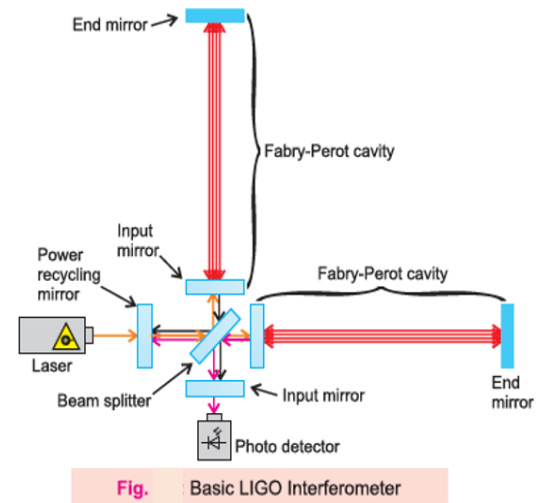
On 14 September 2015, the Universe's gravitational waves were observed for the very first time by LIGO. The gravitational waves which were predicted by Albert Einstein 100 years ago came from a collision between two black holes. It took 1.3 billion years for the waves to arrive at the LIGO detector in the USA. On their work on observation of GWs, Rainer Weiss, Barry C. Barish and Kip S. Thorne received Nobel Prize in 2017. It is interesting to note that one of the team members, Nergis Mavalwala a professor at MIT (Massachusetts Institute of Technology), belongs to Pakistan.

Virgo Detection:

Like LIGO, there is another facility for measuring gravitational waves. This is called Virgo, which works under the European Gravitational Observatory (EGO) Cascina near Pisa, Italy. Virgo is also an interferometer with two arms of 3 km whereas LIGO has 4 km arms. The Virgo Observatory is named after the Virgo constellation, which is visible in the night sky during the months of March, April and May. The Virgo cluster is a group of about 1,500 galaxies about 50 MLYS (Million Light Years) away. Remember one Light Year (LY) is a distance which light travels in one year. The approximate value of $1 \text{ LY} = 9.5 \text{ billion km}$. Virgo has been involved in detecting gravitational wave events, with the first detection in 2017.

8.8 What is meant by optical activity meant? Discuss it.

Ans Optical Activity:

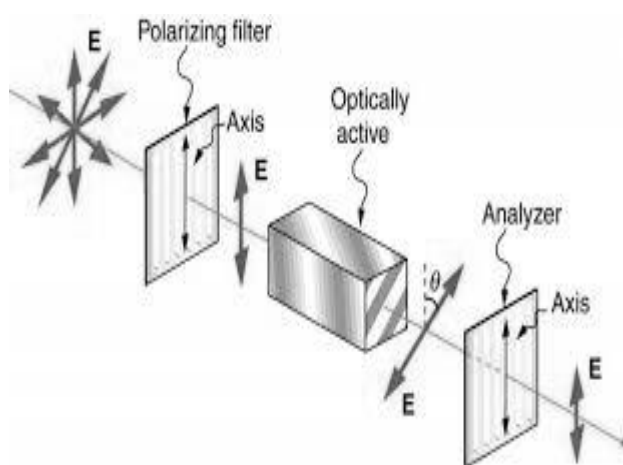


Optical activity is the ability of a substance to rotate the plane of polarization of light passing through it. The rotation is detected with a polarizer or analyzer as shown in Fig. 8.8. Many crystals and solutions rotate the plane of polarization of light passing through them.

Such substances are said to be optically active. Examples are quartz crystals, cinnabar (HgS), sugar water, insulin and collagen. The amount and direction of rotation depends on following factors:

- (i). The type of substance
- (ii). The concentration of the substance (the amount of a substance presentsch quantity of a mixture or solution).
- (iii). The distance the light travels through it.
- (iv). The wavelength of light.

Optical activity occurs due to the asymmetric shape of molecules in the substance, such as being helical. A few millimeters thickness of such crystals will rotate the plane of polarization by many degrees. Certain organic substances, such as sugar and tartaric acid, show optical rotation when they are in a solution. This property of optically active substances can be used to determine their concentration in the solutions.



Numerical Problems

8.1 When an unpolarized light of intensity I_o is incident on a polarizing sheet, find the intensity of light which does not get transmitted.

Given data:

$$\text{Intensity of light (unpolarized)} = I_o$$

$$\text{Intensity of light (transmitted)} = I_t = \frac{I_o}{2}$$

$$\text{Intensity of light (blocked)} = I_b$$

To find:

$$I_b = ?$$

Solution:

$$I_b = I_o - I_t$$

$$I_b = I_o - \frac{I_o}{2}$$

$$I_b = \frac{I_o}{2}$$

8.2 A polarized light beam passes through a polarizer at an angle of 45° . Find the intensity of the transmitted light if the initial intensity is 100 W m^{-2} .

Given data:

$$I_o = 100 \text{ W m}^{-2}$$

$$\theta = 45^\circ$$

To find:

$$I = ?$$

Solution:

$$I = I_o \cos^2 \theta$$

$$I = 100(\cos 45^\circ)^2$$

$$I = 50 \text{ W m}^{-2}$$

8.3 A light wave passes through a polarizer with its electric field aligned at 30° to the horizontal. If the amplitude of the wave is 10 units, what is the amplitude of the wave passing through the polarizer?

Given data:

$$A_o = 10 \text{ units}$$

$$\theta = 30^\circ$$

To find:

$$A = ?$$

Solution:

$$\begin{aligned}A &= A_o \cos \theta \\A &= 10 \cos 30^\circ \\A &= 8.66 \text{ units}\end{aligned}$$

8.4 What angle is required between the direction of polaroid light and the axis of a polaroid filter to reduce its intensity by 85%?

Given data: Intensity of light (blocked) = $I_b = 85\%$ of $I_o = 0.85 I_o$
Intensity of light (transmitted) = $I = (100 - 85)\%$ of $I_o = 0.15 I_o$

To find: $\theta = ?$

Solution:

$$\begin{aligned}I &= I_o \cos^2 \theta \\0.15 I_o &= I_o \cos^2 \theta \\\cos^2 \theta &= 0.15 \\\cos \theta &= \sqrt{0.15} = 0.3873 \\\theta &= \cos^{-1}(0.3873) = 67.2^\circ\end{aligned}$$

8.5 An unpolarized light having intensity of 15 W m^{-2} is incident on a pair of polarizers. The first polaroid filter has its transmission axis at 50° from the vertical. The second polaroid filter has its transmission axis at 20° from the vertical. Calculate the intensity of light transmitted to both filters.

Given data:

Intensity of light (unpolarized) = $I_o = 15 \text{ W m}^{-2}$
angle (1st polarizer) = $\theta_1 = 50^\circ$
angle (2nd polarizer) = $\theta_2 = 20^\circ$

To Find:

Intensity (1st polarizer) = $I_1 = ?$
Intensity (2nd polarizer) = $I_2 = ?$

Solution:

(a)

$$\begin{aligned}I_1 &= I_o - I_b \\I_1 &= I_o - \frac{I_o}{2} = \frac{I_o}{2} \\I_1 &= 15 \text{ W m}^{-2} = 7.5 \text{ W m}^{-2}\end{aligned}$$

(b)

Angle between the two polarizer = θ

$$\begin{aligned}\theta &= \theta_1 - \theta_2 \\I_2 &= I_1 \cos^2 \theta \\I_2 &= 7.5 (\cos 30^\circ)^2 \\I_2 &= 7.5 (0.866)^2 \\I_2 &= 5.625 \text{ W m}^{-2}\end{aligned}$$

8.6 Two polarizing sheets have their polarizing directions parallel so that intensity of emitted light is maximum. Through what angle must either sheet be rotated if the intensity is to be dropped by half?

Given data:

Intensity of light I_o
Intensity of transmitted light = $I = \frac{I_o}{2}$

To find:

$\theta = ?$

Solution:

$$\begin{aligned}I &= I_o \cos^2 \theta \\\frac{I_o}{2} &= I_o \cos^2 \theta\end{aligned}$$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

8.7 We wish to use a glass plate of refractive index of 1.5 in air as a polarizer. Find the polarizing angle and angle of refraction.

Given data: $n_{\text{glass}} = 1.5$

$$n_{\text{air}} = 1$$

To Find: $\theta_p = ?$

$$\theta_r = ?$$

Solution: $\tan \theta_p = n_{\text{glass}}$
 $\theta_p = \tan^{-1}(n_{\text{glass}})$
 $\theta_p = \tan^{-1}(1.5) = 56.3^\circ$
 $\theta_r = 90^\circ - \theta_p$
 $\theta_r = 90^\circ - 56.3^\circ = 33.7^\circ$

8.8 At what angle of incidence will light reflected from water be completely polarized?

Given data: $n_{\text{water}} = 1.33$

To Find:

$$\theta_p = ?$$

Solution: $\tan \theta_p = n_{\text{water}}$
 $\theta_p = \tan^{-1}(n)$
 $\theta_p = \tan^{-1}(1.33) \approx 53^\circ$

8.10 A polarizer and an analyzer have their axes aligned at 60° . What is the fraction of the initial intensity that emerges?

Given data: $\theta = 60^\circ$

$$\text{Intensity after polarization} = I_o$$

To Find: $\frac{I}{I_o} = ?$

Solution: $I = I_o \cos^2 \theta$
 $I = I_o \cos^2(60^\circ)$
 $\frac{I}{I_o} = 0.25$

8.11 If the gravitational waves have a wavelength of 3000 km, then find their frequency assuming it moves with the speed of light?

Given data:

$$\lambda = 3000 \text{ km} = 3.0 \times 10^6 \text{ m}$$

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

To Find: $f = ?$

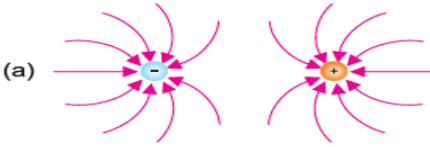
Solution: $c = f\lambda$

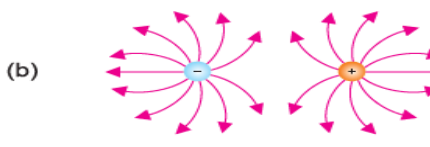
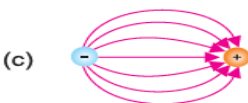
$$f = \frac{c}{\lambda}$$

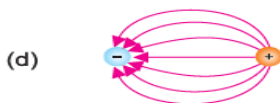
$$f = \frac{3 \times 10^8}{3 \times 10^6} = 100 \text{ Hz}$$

Chapter 9 Electrostatics and Current Electricity

Exercise MCQs

- 9.1** Two-point charges A and B are separated by 10 m. If the distance between them is reduced to 5 m, the force exerted on each:
- decreases to half its original value
 - increases to twice the original value
 - decreases to one quarter of its original value
 - increases four times to its original value
- 9.2** Which electric charge is possible on a particle?
- 2.5×10^{-19} C
 - 3.2×10^{-19} C
 - 1.6×10^{-19} C
 - 6.02×10^{23} C
- 9.3** Which diagram best represents the electric field lines around two oppositely charged particles?
- (a) 

(b) 
- (c) 

(d) 
- 9.4** What is the work done on an electron by potential difference of 100 volts?
- 1.6×10^{-19} eV
 - 1.6×10^{-17} eV
 - 6.25×10^{-17} eV
 - 100 eV
- 9.5** The potential at a point situated at a distance of 50 cm from a charge of 50 μ C is:
- 9×10^{-4} volts
 - 18×10^{-4} volts
 - 9×10^5 volts
 - 18×10^4 volts
- 9.6** A ball of weight 0.1 N having a charge of 100 μ C remained suspended between two oppositely charged horizontal metal plates. The electric intensity between the plates is:
- 10 N C^{-1}
 - 100 N C^{-1}
 - 1000 N C^{-1}
 - 10000 N C^{-1}
- 9.7** A piece of wire has resistance of 4 Ω . It is doubled on itself so that its length becomes half but area of cross-section is doubled. Its resistance now will be:
- 8 Ω
 - 4 Ω
 - 2 Ω
 - 1 Ω
- 9.8** The current through a conductor is 3.0 A when it is attached across a potential difference of 6.0 V. How much power is used?
- 0.5 W
 - 2.0 W
 - 9.0 W
 - 18 W
- 9.9** The algebraic sum of potential changes for a complete circuit is zero. It is the statement of:
- Ohm's law
 - Gauss's law
 - Kirchhoff's 1st law
 - Kirchhoff's 2nd law
- 9.10** The radius of curvature of the path of a charged particle in a uniform magnetic field is directly proportional to:
- the particle's charge
 - the particle's momentum
 - the particle's energy
 - the flux density of the field

Answer Key					
Q.No.	Option	Q.No.	Option	Q.No.	Option
9.1	D	9.5	C	9.9	D
9.2	B & C	9.6	C	9.10	B
9.3	D	9.7	D		
9.4	D	9.8	D		

Short Answer Questions

- 9.1** How does a moving conductor like an aeroplane acquire charge as it flies through the air? Describe briefly.

Ans When aeroplane moves through the air with high speed, it acquires charge due to friction with air molecules and dust particles. This causes transfer of electrons between aeroplane and air. The transfer

of electrons creates a net positive or negative charge on the aeroplane surface resulting in a buildup of static electricity.

9.2 Define electric intensity and electric potential.

Ans Electric Intensity (Electric Field Strength):

Electric field strength or electric field intensity at any point is defined as the electric force experienced by a unit positive charge q placed at that point. It is a vector quantity.

$$\vec{E} = \frac{\vec{F}}{q}$$

Units: S.I unit of electric intensity is N C^{-1} or V m^{-1} .

Electric Potential: The electric potential at any point is equal to the work done per unit charge in bringing a unit positive charge from infinity to that point keeping it in electrostatic equilibrium. It is a scalar quantity.

$$V = \frac{W}{q}$$

Units: S.I unit of electric potential is J C^{-1} or V .

9.3 A battery is rated at 100 A h (ampere-hour). How much charge can this battery supply?

Ans Given: $I = 100 \text{ A}$ and $t = 1 \text{ h} = 3600 \text{ s}$

We know that $Q = I \times t$

$$Q = 100 \times 3600$$

$$Q = 360,000 \text{ C} = 3.6 \times 10^5 \text{ C}$$

9.4 Is electron-volt a unit of potential difference or energy? Explain.

Ans The electron volt (eV) is a smaller unit of energy used in atomic scale.

By definition one electron-volt is “the amount of energy acquired or lost by electron as it traverses a potential difference of one volt.”

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

9.5 A copper wire of length L has resistance R . It is stretched to double of its length. What will be the resistance of the new length of wire?

Ans The resistance (R) of the wire is given by

$$R = \rho \frac{L}{A}$$

where, ρ = resistivity; L = length of wire; and A = cross – sectional area of wire

When wire stretches to double of its length, then area A becomes half. Therefore, new resistance R' is given as

$$R' = \rho \frac{L'}{A'} = \rho \left(\frac{2L}{A/2} \right)$$

$$R' = 4 \left(\rho \frac{L}{A} \right)$$

$$R' = 4R$$

Thus, new resistance will be 4 times the original resistance.

9.6 Why does the resistance of a conductor rise with increase in temperature?

Ans The resistance offered by a conductor to the flow of current is due to collisions. As the temperature of conductor rises, the amplitude of vibration of the atoms in the lattice increases. The probability of collision of free electrons with lattice atoms also increases. Hence, the resistance of conductor increases.

9.7 Is the filament resistance lower or higher in a 500 W-220 V light bulb than in a 100 W-220 V bulb?

Ans According to the formula:

$$R = \frac{V^2}{P}$$

a. **For 500 W bulb:**

$$a. R_1 = \frac{220^2}{500} = 96.8 \Omega$$

b. **For 100 W bulb:**

$$R_2 = \frac{220^2}{100} = 484 \Omega$$

Thus, the 500 W filament has lower resistance than the 100 W filament.

9.8 Why does resistance of a thermistor changes as temperature increases?

Ans Most thermistors have negative temperature coefficient i.e., their resistance decreases as temperature increases. This is because increasing temperature provides more energy to the charge carriers (electrons or holes), enabling them to move more freely and thus reducing resistance.

9.9 Which materials can be used to construct Faraday's cage and why?

Ans Conductors can be used to construct Faraday's cage.

The principle of Faraday's cage demands a material that contains a lot of free electrons that can move freely to the surface of the material. Only the conductors, like copper, aluminum, etc., have free electrons. So, conductors are used to construct Faraday's cage.

Constructed Response Questions

9.1 Electric lines of force never cross each other. Why?

Ans Electric field E has only one direction at a given point, if the electric lines cross each other then, at the point of intersection E would have more than one direction. This is physically not possible. Hence electric field lines can never cross each other.

9.2 Is E necessarily zero inside a charged rubber balloon if the balloon is spherical? Assume that charge is distributed uniformly over the surface.

Ans Yes, E is zero inside a spherical charged rubber balloon.

If we consider Gaussian surface inside the charged rubber balloon, then charged enclosed by this surface is zero.

$$\text{As } \Phi = \frac{q}{\epsilon_0}$$

$$\Phi = \frac{0}{\epsilon_0} = 0 \text{ -----(1)} \quad (\because q = 0)$$

$$\text{Also } \Phi = E.A \text{ -----(2)}$$

Comparing equation (1) and (2) we get

$$E.A = 0$$

since A cannot be zero hence electric intensity E will be zero.

9.3 Electrostatic force is 10^{38} times stronger than gravitational force. Argue that our galaxy should be almost electrically neutral.

Ans If galaxies carried net electric charge, then electrostatic force would cause massive repulsion between stars and planets. So, planetary orbits would not be stable and hence it would prevent the formation of the galaxies. Nature must balance the charges so that large scale structures would exist. Thus, galaxies are expected to be almost electrically neutral.

9.4 An uncharged conducting hollow sphere is placed in the field of a positive charge q . What will be the net flux through the shell?

Ans The net electric flux through the shell is zero.

According to the Gauss's law, net electric flux through the closed surface is given by:

$$\phi = \frac{q}{\epsilon_0}$$

For an uncharged conducting hollow sphere, $q = 0$,

$$\phi = \frac{0}{\epsilon_0} = 0$$

Hence, net flux through the shell remains zero.

9.5 A potential difference is applied across the ends of a copper wire. What is the effect on the drift velocity of free electrons by (i) increasing the potential difference? (ii) decreasing the length and the temperature of the wire?

i) The drift velocity of free electrons increases by increasing the potential difference.

When potential difference is increased, then the rate of flow of charges increases. Hence drift velocity of free electrons will also increase.

ii) The drift velocity of free electrons increases by decreasing the length and temperature of wire.

By decreasing the length and temperature, the resistance of wire will decrease. Hence the drift velocity of the free electron will increase.

9.6 Why the terminal potential difference of a battery decreases when the current drawn from it is increased?

Ans The relation between terminal potential difference and emf is given by

$$E = V_t + I r$$

or,

$$V_t = E - I r$$

where V_t is terminal potential difference, E is emf and r is internal resistance of battery.

From the above relation it is clear that when current is drawn from the battery, I will increase and the terminal potential difference V_t will decrease.

Comprehensive Questions

9.1 Explain the electric potential and prove that electric field intensity is equal to the negative of potential gradient.

Ans: Electric Potential:

Electric potential at a point in electric field is equal to the work done in bringing a unit positive charge from infinity to that point against the electric field keeping it in equilibrium.

Mathematically, $V = \frac{W}{q}$

Explanation:

Consider a positive charge q which is allowed to move in the uniform electric field between two oppositely charged plates A and B. The charge moves from +ve plate B towards the -ve plate A due to the electric force $\vec{F} = q_0 \vec{E}$ and gains K.E. If the charge is to be moved from plate A towards plate B with constant velocity, an external force equal and opposite to $q_0 \vec{E}$ is needed. Then the charge will gain electric P.E.

It means that the work done in moving the charge from plate A towards B appears as the increase in its potential energy i.e.,

$$\text{Change in P.E} = \text{Work done from A to B}$$

$$U_B - U_A = W_{AB}$$

Potential Difference:

The potential difference between two points A and B in electric field is defined as the work done in carrying a unit positive charge from A to B while keeping the charge in equilibrium i.e.,

$$\Delta V = \frac{W_{AB}}{q}$$

or $V_B - V_A = \frac{\Delta U}{q} = \frac{W_{AB}}{q} \text{-----(1)}$

Thus, the potential difference can also be defined as the difference of potential energy per unit charge.

Unit:

The unit of potential difference is volt.

Potential difference between two points is 1V, if work done in moving unit positive charge from one point to another point, keeping the charge in equilibrium, is one joule.

Mathematically, $1V = \frac{1J}{1C}$

Absolute Potential:

To find the electric or absolute potential at a point in field, we must have a reference point which is at infinity and has zero electric potential. If we take point A to be at infinity then $V_A \approx 0$, and electric potential will be

$$V_B = \frac{W_{AB}}{q}$$

Generally, $V = \frac{W}{q}$

Electric Potential as a potential gradient:

From equation (1), we have

$$\Delta V = \frac{W}{q}$$

$$\Delta V = \frac{\vec{F} \cdot \vec{d}}{q_0}$$

$$\Delta V = \frac{Fd \cos \theta}{q_0} = \frac{q_0 E \Delta r \cos(180)^\circ}{q_0}$$

$$\Delta V = -E \Delta r$$

$$E = -\frac{\Delta V}{\Delta r}$$

Hence it is proved that electric field intensity is negative of potential gradient.

9.1A State and explain Coulomb's law. Also, describe the effect of medium between two charges on Coulomb's force between them.

Ans Statement:

The force between two-point charges is directly proportional to the product of magnitudes of charges and inversely proportional to the square of the distance between them.

Explanation:

Consider two-point charges q_1 & q_2 placed at a distance 'r' from each other. The magnitude of mutual force according to coulomb's law is.

$$F \propto q_1 q_2 \quad \text{----- (i)}$$

$$F \propto \frac{1}{r^2} \quad \text{----- (ii)}$$

Combining (i) and (ii)

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

k = constant of proportionality called electrostatic constant or Coulomb's constant.

Dependence of k:

1. Nature of medium between charges
2. The system of units in which F, q and r, are measured

Unit of k: $\text{N m}^2 \text{C}^{-2}$

Mutual Force: $\vec{F}_{21} = -\vec{F}_{12}$

Coulomb's force is a mutual force, it means that if q_1 exerts a force on q_2 then q_2 also exerts an equal and opposite force on q_1 . Let

$\vec{F}_{21}$ = Force exerted on q_2 by q_1

$\vec{F}_{12}$ = Force exerted on q_1 by q_2

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad \text{----- (v)}$$

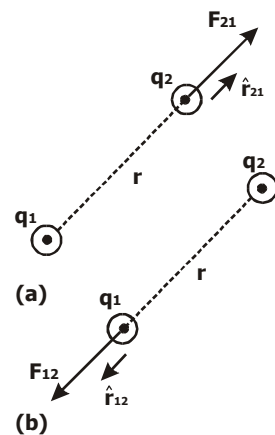
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \text{----- (vi)}$$

Eq.(v) becomes

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} (-\hat{r}_{12}) \quad \text{as } \hat{r}_{21} = -\hat{r}_{12}$$

$$\vec{F}_{21} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

$$\vec{F}_{21} = -\vec{F}_{12}$$



Effect of medium on Coulomb's Force:

Case- 1:

When medium between charges is free space / vacuum.

$$F = k \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \text{-----(iii)}$$

Permittivity of free space:

It is an ability of a medium to allow the electric lines of force to pass through it.

ϵ_0 = An electrical constant, called permittivity of free space.

$$\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2}$$

$$k = \frac{1}{4\pi\epsilon_0} = \frac{1}{4 \times 3.14 \times 8.85 \times 10^{-12}} = 9 \times 10^9 \text{Nm}^2 \text{C}^{-2}$$

In vector form

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

Case- 2:**When medium between charges is an insulator**

If the medium between charges is an insulator, then the electrostatics force decreases by a constant factor called relative permittivity of medium (ϵ_r). It is different for different materials. Coulomb's force when there is a dielectric placed between the charges is given by

$$F' = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} \quad F' = \frac{1}{\epsilon_r} \left(\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \right) \text{----- (iv)}$$

Definition of ϵ_r

Putting the value of F from eq. (iii) in eq. (iv) we get

$$F' = \frac{1}{\epsilon_r} F = \frac{F}{\epsilon_r} \quad \text{where} \quad \epsilon_r > 1 \text{ (For all insulators)}$$

$$\text{Hence } F' < F$$

$$\epsilon_r = \frac{F}{F'}$$

Relative permittivity is ratio of force between the two charges when there is a free space between them to the force when a dielectric is used as a medium between them. It has no units.

9.1B Find the electric flux through a surface enclosing a charge.

Ans Consider a closed surface in the shape of a sphere.

r = radius of the sphere

$+q$ = charge placed at centre of the sphere.

To apply $\phi_e = \vec{E} \cdot \vec{A}$, the surface area should be flat. So, divide the surface area of sphere into n small patches. Each patch has flat area like $\Delta \vec{A}_1, \Delta \vec{A}_2, \Delta \vec{A}_3 \dots \dots \dots \Delta \vec{A}_n$.

The direction of each vector area is along a perpendicular drawn outward to the patches.

The electric intensities at the centers of the vector areas $\Delta \vec{A}_1, \Delta \vec{A}_2, \Delta \vec{A}_3 \dots \dots \dots \Delta \vec{A}_n$ are

$$\vec{E}_1, \vec{E}_2, \vec{E}_3 \dots \dots \dots \vec{E}_n$$

$$\vec{E}_1, \vec{E}_2, \vec{E}_3 \dots \dots \dots \vec{E}_n$$

The total flux passing through the closed surface is

$$\phi_e = \phi_1 + \phi_2 + \phi_3 \dots \dots \dots + \phi_n$$

$$\phi_e = \vec{E}_1 \cdot \Delta \vec{A}_1 + \vec{E}_2 \cdot \Delta \vec{A}_2 + \vec{E}_3 \cdot \Delta \vec{A}_3 \dots \dots \dots + \vec{E}_n \cdot \Delta \vec{A}_n$$

For each patch, $\vec{E}$ and $\Delta \vec{A}$ are parallel, so $\theta = 0^\circ$

$$\phi_e = E_1 \Delta A_1 \cos 0^\circ + E_2 \Delta A_2 \cos 0^\circ + E_3 \Delta A_3 \cos 0^\circ \dots \dots \dots + E_n \Delta A_n \cos 0^\circ$$

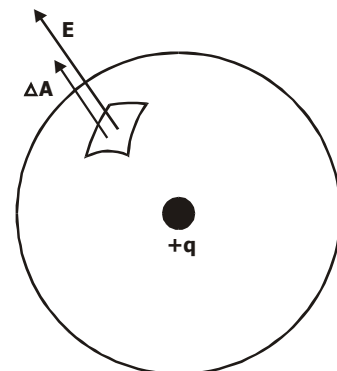
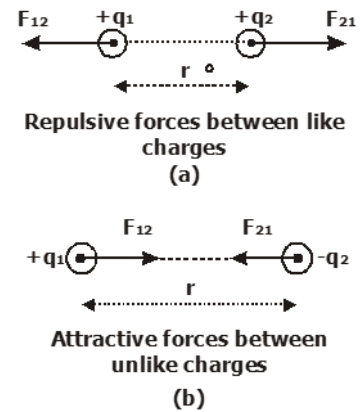
$$\phi_e = E_1 \Delta A_1 + E_2 \Delta A_2 + E_3 \Delta A_3 + \dots \dots \dots E_n \Delta A_n \quad (\cos 0^\circ = 1)$$

Because of spherical symmetry

$$E_1 = E_2 = E_3 = \dots \dots \dots E_n = E$$

$$\text{Where } E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\phi_e = E \Delta A_1 + E \Delta A_2 + E \Delta A_3 + \dots \dots \dots + E \Delta A_n$$



The total electric flux through the surface of the sphere due to a charge q at its centre is q/ϵ_0 .

$$\begin{aligned}\phi_e &= E(\Delta A_1 + \Delta A_2 + \Delta A_3 + \dots + \Delta A_n) \\ \phi_e &= E(\text{Total area of the spherical surface}) \\ \phi_e &= E(4\pi r^2) \\ \phi_e &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (4\pi r^2) \\ \phi_e &= \frac{q}{\epsilon_0} \\ \phi_e &= \frac{1}{\epsilon_0} q\end{aligned}$$

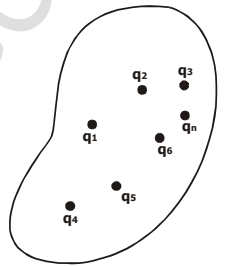
Now consider a closed surface 'S' around the sphere as shown in Fig. Flux through the closed surface S is the same as that through the sphere.

9.1C State and prove Gauss's law. Also apply it to find electric field intensity due to the field of charged conducting sphere.

Ans Gauss's law

Statement: The flux through any closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed in it.

$$\begin{aligned}\phi &= \frac{1}{\epsilon_0} Q \\ \phi &= \frac{Q}{\epsilon_0}\end{aligned}$$



Proof:

Consider charges $q_1, q_2, q_3 \dots q_n$ which are present in a closed surface as shown in Fig.

Consider a closed surface around each charge which is called as Gaussian surface. The flux passing through each Gaussian surface is given by

$$\begin{aligned}\phi_1 &= \frac{q_1}{\epsilon_0} \\ \phi_2 &= \frac{q_2}{\epsilon_0} \\ \phi_3 &= \frac{q_3}{\epsilon_0} \\ &\dots\dots\dots \\ &\dots\dots\dots \\ &\dots\dots\dots \\ \phi_n &= \frac{q_n}{\epsilon_0}\end{aligned}$$

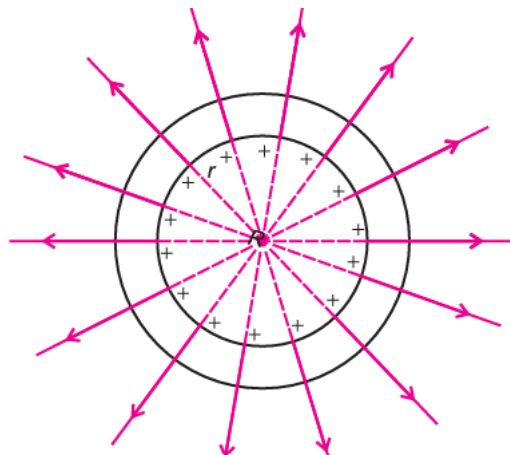
Total flux through the closed surface is

$$\begin{aligned}\phi_e &= \phi_1 + \phi_2 + \phi_3 + \dots + \phi_n \\ \phi_e &= \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \frac{q_3}{\epsilon_0} \dots + \frac{q_n}{\epsilon_0} \\ \phi_e &= \frac{1}{\epsilon_0} (q_1 + q_2 + q_3 + \dots + q_n) \\ \phi_e &= \frac{1}{\epsilon_0} (\text{Total charge enclosed by the closed surface}) \\ \phi_e &= \frac{Q}{\epsilon_0} (\because Q = q_1 + q_2 + q_3 + \dots + q_n)\end{aligned}$$

So, the flux through a closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed in it.

Field due to charged conducting sphere:

Consider a conducting sphere of radius R containing a charge q . We know that all the charge is distributed uniformly over the surface of sphere as shown in Fig. We can also conclude from the spherical symmetry that the electric field is radial everywhere and that its magnitude depends only on the distance r from the centre of the sphere. Thus, the magnitude E is uniform over a spherical surface with any radius r concentric with the spherical conductor. Therefore, we take our Gaussian surface as an imaginary sphere with radius r greater than the radius R of the conducting sphere.



The area of the Gaussian sphere is $4\pi r^2$, and because E is uniform over the sphere, the total flux through the whole surface will be:
Electric flux

$$\Phi = EA = E \times 4\pi r^2 \text{-----(i)}$$

$$\text{By Gauss law } \Phi = \frac{q}{\epsilon_0} \text{-----(ii)}$$

Comparing equation i and ii

$$\frac{q}{\epsilon_0} = E \times 4\pi r^2$$

$$E = \frac{q}{\epsilon_0} \times \frac{1}{4\pi r^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

In vector form

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \hat{r}$$

9.2 State and explain Kirchhoff's rules.

Ans: **Kirchhoff's First Rule (KCL)**

Statement: The sum of all the currents meeting at a point in the circuit is zero.

Mathematical form:

$$\sum I = 0$$

Explanation:

Consider a circuit where four wires meet at point A as shown in fig. The currents I_1 and I_2 are flowing towards point A and currents I_3 and I_4 are flowing away from point A.

According to the convention current flowing towards a point is taken as positive and that flowing away from a point is taken as negative. Apply Kirchhoff's 1st rule, we have

$$I_1 + I_2 = I_3 + I_4$$

$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$

$$\sum I = 0$$

Kirchhoff's 1st rule is the manifestation (explanation) of law of conservation of charge. If there is no sink or of charge at the point, the total charge flowing towards a point must be equal to the total charge flowing away from the point

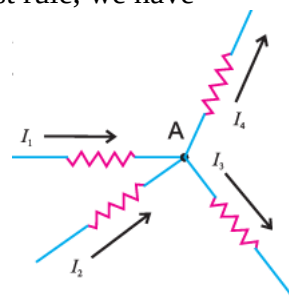
Kirchhoff's Second Rule (KVL)

Statement: The algebraic sum of voltages changes in a closed circuit or a loop must be equal to zero.

Mathematical form:

$$\sum V = 0$$

Explanation:



Consider a closed circuit as shown in figure. The direction of current I flowing through the circuit depends upon the cell having the greater emf. Suppose E_1 is greater than E_2 . So, the current flows in anti-clockwise direction. When a positive charge ΔQ due to the current I in the closed circuit, passes through the cell E , from low to high potential, it gains energy, because the work is done on it. The gained energy is $E_1 \Delta Q$

When the current passes through the cell E_2 it loses energy, equal to $-E_2 \Delta Q$ because here charge passes from high potential to low potential. In going through the resistor R_1 , the charge ΔQ loses energy equal to $-IR_1$. Similarly the loss of energy while passing through the resistor R_2 is $-IR_2$. Finally, the charge reaches negative terminal of the cell E , from where we started. According to the law of conservation of energy the change in energy of our system is zero. So, we can write

$$E_1 - IR_1 - E_2 - IR_2 = 0$$

$$\sum V = 0$$

Kirchhoff's 2nd rule is the manifestation (explanation) of law of conservation of energy.

9.2A Explain Resistivity and conductance what is effect of temperature on resistance and resistivity. Also describe temperature co-efficient of resistance and resistivity. Derive their formula.

Ans The resistance R of a wire is directly proportional to its length L and inversely proportional to the area of cross section 'A'

$$R \propto L \quad (1)$$

$$R \propto \frac{1}{A} \quad (2)$$

Combining (1) and (2)

$$R \propto \frac{L}{A}$$

$$R = \text{Constant} \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$

$$\rho = \frac{RA}{L}$$

ρ is called resistivity or specific resistance.

Resistivity: ρ

Resistance of (1 m^3) one-meter cube of a material is called resistivity.

$$\text{Units of } \rho = \frac{\text{ohm} \times \text{m}^2}{\text{m}}$$

$$\rho = \text{ohm} \times \text{m}$$

$$\rho = \Omega \text{m}$$

Resistivity and its dependence upon temperature:

Resistivity of a material depends upon its temperature. The resistance offered by a conductor to the flow of electric current is due to collision of electrons with atoms of lattice. As temperature of conductor rises the amplitude of vibration of atoms increase and chances of their collision with free electrons also increase. Hence resistance of conductor increases.

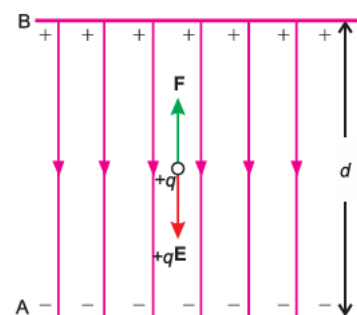
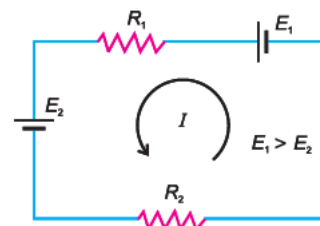
Temperature coefficient:

Experimentally it is found that change of resistance of a metallic conductor is nearly linear over a wide range of temperature above and below 0°C . See Fig. Over such a range.

Fractional change in resistance per Kelvin is known as temperature Co-efficient of resistance.

It is denoted by α .

$$\alpha = \frac{R_t - R_0}{R_0 t}$$



Proof:

Let

R_0 = Resistance of conductor at 0°C

R_t = Resistance of conductor at $t^\circ\text{C}$

t = Change in temperature

$R_t - R_0$ = Change in Resistance.

Experimentally

$$R_t - R_0 \propto R_0 \quad (\text{i})$$

$$R_t - R_0 \propto t \quad (\text{ii})$$

Combining eqs. (i) and (ii)

$$R_t - R_0 \propto R_0 t$$

$$R_t - R_0 = \text{constant} \times R_0 t$$

$$R_t - R_0 = \alpha R_0 t$$

$$\alpha = \frac{R_t - R_0}{R_0 t}$$

$$\alpha = \frac{R_t - R_0}{R_0 t} \quad (\text{iii})$$

α = Fractional change in resistance per kelvin

Units

Units of α is K^{-1}

As we know that $\rho = \frac{RA}{L}$

Resistivity is directly proportional to resistance R . So, we can write eq (iii) in terms of resistivity as:

$$\alpha = \frac{\rho_t - \rho_0}{\rho_0 t}$$

Conductance:

Reciprocal of resistance is called conductance

$$\text{Conductance} = \frac{1}{\text{Resistance}}$$

Unit of conductance are ohm^{-1} or mho or "siemen"

Conductivity:

Reciprocal of resistivity is called conductivity

$$\sigma = \frac{1}{\rho}$$

Unit of

$$\sigma = \frac{1}{\text{ohm} \times \text{m}}$$

= $\text{ohm}^{-1} \text{m}^{-1}$

9.2B What do you mean by electrical power? Derive an expression for the power dissipated in a resistor.

Ans Electrical Power:

The rate at which the battery is supplying electrical energy is the power output or electrical power of the battery.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

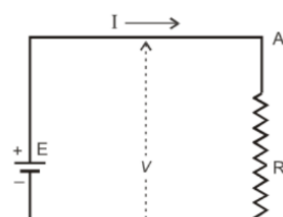
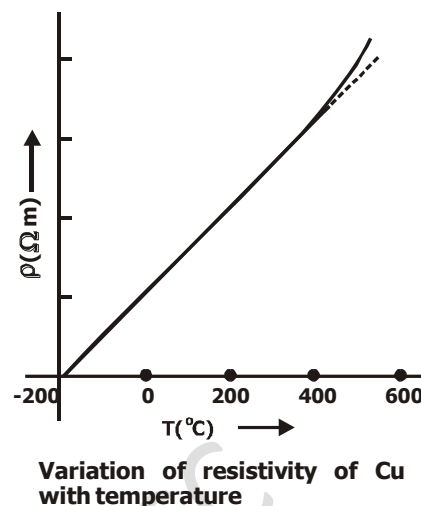
Explanation:

Consider a circuit in which a battery is connected in series with a resistance R . A steady current I flow through the circuit and steady potential difference V exists between the terminals A and B of resistor 'R'. Terminal 'A' is higher potential than terminal 'B'.

By definition, Potential Difference is amount of work done in moving a charge ΔQ up through potential difference V , is given by

$$V = \frac{\Delta W}{\Delta Q}$$

Work done = $\Delta W = V \times \Delta Q$ the work done is equal to the energy supplied by battery.

Power dissipated in resistors:

By definition of power:

$$\text{Electrical power} = \frac{\text{Energy supplied}}{\text{Time}}$$

$$\text{Electrical power} = \frac{\text{Work done}}{\text{Time}}$$

$$\text{Electrical power} = \frac{\Delta W}{\Delta t}$$

$$\text{Electrical power} = \frac{V \Delta Q}{\Delta t}$$

$$\text{Electrical power} = V \times I$$

$$\text{Electrical Power: } V \times I$$

By law of conservation of energy

Power supplied by battery = power dissipated in the resistor R

$$\text{Power dissipated} = P = V \times I \text{ --- (i)}$$

By OHM'S LAW

$$V = IR$$

$$P = IR \times I$$

$$P = I^2 R \text{ --- (ii)}$$

$$\text{as } I = \frac{V}{R}$$

Putting the value in (i)

$$P = V \times I$$

$$P = V \times \frac{V}{R}$$

$$P = \frac{V^2}{R}$$

$$P = VI = I^2 R = \frac{V^2}{R} \text{ --- (iii)}$$

Eq. (i), (ii), and (iii) are used for calculating the power dissipated in a resistor.

Unit of power is "watt"

9.2C Define and explain term emf and potential difference.

Ans Electromotive Force:

The emf E is the energy supplied to unit charge by the cell.

Formula:

$$E = \frac{\Delta W}{\Delta Q}$$

Units:

Unit of emf are volt

$$V = JC^{-1}$$



Electromotive force of a cell

Explanation:

When a source of electrical energy, e.g. a cell or battery is connected across a resistance R, it maintains a steady current through it. The cell supplies energy which is used in the resistance of circuit. Suppose a charge ΔQ passes through circuit in time Δt . This charge enters the cell at its lower potential end and leaves at its high potential end. The source supplies energy = Δw to the positive charge to force it to go to the potential of high potential. as emf is the energy supplied to unit charge by the cell

Internal Resistance:

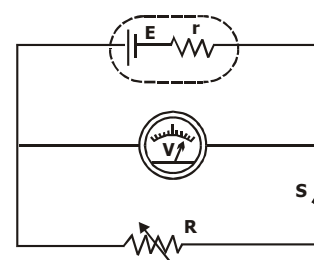
A cell offers some resistance. This resistance is due to electrolyte present between the two electrodes of cell. It is called internal resistance r of the cell.

$$I = \frac{E}{R + r}$$

$$E = I(R + r)$$



An equivalent circuit of a cell of emf E and internal resistance r.



The terminal potential difference V of a cell is $E - IR$

$$E = IR + Ir$$

$IR = V$ is terminal potential difference of cell when current I is flowing through circuit.

$$E = V + Ir$$

$$V = E - Ir$$

Case-I:

When switch S is open, no current passes through resistance $I = 0$

$$V = E - Ir = E - r(0) = E \quad \text{i.e.} \quad V = E$$

Case-II:

when switch S is closed (on), current I is passing through circuit.

$$V = E - Ir$$

$$V < E$$

Terminal voltage V in the presence of the current I is less than emf E by Ir .

Note

- (i) The emf is the cause and potential difference is its effect.
- (ii) The emf is always present even when no current is drawn through the battery or the cell:
- (iii) But the potential difference 'V' across conductor is zero when no current flows through it.

9.3 What is a Wheatstone bridge? Explain its working with help of a diagram.

Ans Wheatstone bridge:

A circuit which is used to find the unknown resistance of wire is called Wheatstone bridge.

Explanation:

The Wheatstone bridge is a circuit shown in Fig. It consists of four resistances R_1 , R_2 , R_3 and R_4 they are connected in such a way that circuit forms a mesh ABCDA. A battery is connected between points A and C galvanometer of resistance R_g is connected between points B and D. When switch S is closed, a current flows through galvanometer. To apply Kirchhoff's 2nd rule, we consider three loops ADBA, BCDB and ADCA. We assume three currents I_1 , I_2 and I_3 flowing through loops ADBA, BCDB and ADCA respectively in anticlockwise direction.

Apply Kirchhoff's 2nd rule to loop ADBA.

$$-I_1 R_1 - (I_1 - I_2) R_g - (I_1 - I_3) R_3 = 0 \quad \text{--- (i)}$$

Apply Kirchhoff's 2nd rule to loop BCDB.

$$-I_2 R_2 - (I_2 - I_3) R_4 - (I_2 - I_1) R_g = 0 \quad \text{--- (ii)}$$

The current flowing through galvanometer will be zero if

$$I_1 - I_2 = 0 \text{ or } I_1 = I_2 \quad (\text{Null condition})$$

With this condition eq(i) and eq(ii) become

$$-I_2 R_1 - (0) R_g - (I_2 - I_3) R_3 = 0$$

$$-I_2 R_1 = (I_2 - I_3) R_3 \quad \text{--- (iii)}$$

$$-I_2 R_2 - (I_2 - I_3) R_4 - (0) R_g = 0$$

$$-I_2 R_2 = (I_2 - I_3) R_4 \quad \text{--- (iv)}$$

Dividing eq (iii) by eq (iv) we get

$$\frac{-I_2 R_1}{-I_2 R_2} = \frac{-(I_2 - I_3) R_3}{-(I_2 - I_3) R_4}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$R_4 = \frac{R_2}{R_1} \times R_3$$

We can find unknown resistance R_4 by replacing the values on right hand side of the equation.

9.4 What is a light dependent resistor (LDR)? How can this be used as ON-OFF switch for lighting?

Ans Light dependent resistor (LDR) is a resistor whose resistance decreases with increasing light intensity due to this property it is also called photo resistor.

Construction:

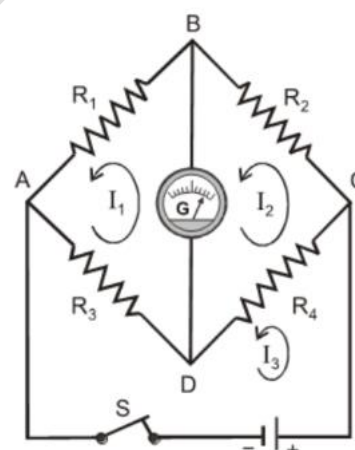


Fig. Wheatstone bridge circuit.

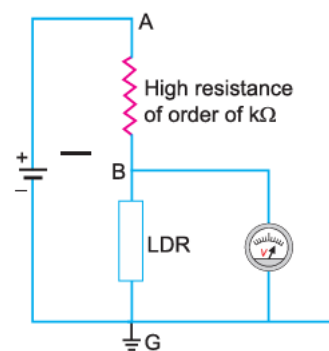
LDRs are typically made from semiconductor material like cadmium sulphide. The material is deposited in a special pattern on an insulating plate.

Working Principle:

The principle used in an LDR is the increase in the conductivity of the material on exposing it to light. In darkness, the semiconductor material has a few free electrons resulting in high resistance. When light photons hit the material, they transfer energy to electrons in the outer orbits, thus making free electrons to conduct electricity. This decreases the resistance of LDR. This change in resistance can be measured and used in circuits to sense light levels.

An LDR works just like a switch that turns ON at dusk and OFF at dawn.

LDRs are commonly used in light sensing circuits such as automatic lighting systems in homes and streets. In a typical circuit, an LDR can be a part of voltage divider, that converts the resistance change into measurable voltage change. This voltage can then be read by a micro-controller or other control circuitry perform actions based on light levels. A circuit is shown in Fig. in which an LDR is used as voltage divider. In the dark, the LDR has a very high resistance as compared to the standard resistance in the provided circuit. Therefore, the voltage drops across the LDR are very large. When LDR is exposed to light, the resistance of LDR decreases to very low. Now, the voltmeter registers a lower reading. Hence, the change in light intensity gives rise to change in desired voltage. Therefore, by connecting mid-point B to a NOT gate, LDR can be used as a switch.



9.5 What is a potentiometer? Describe its working.

Ans Potentiometer:

An instrument which can measure and compare potential differences accurately is a Potentiometer. Potentiometers are one of the most accurate methods for measuring potential.

Explanation:

A voltmeter measures potential difference across a resistor. It cannot measure voltage accurately because it draws some current from the circuit. An ideal voltmeter has an infinite resistance, so that it cannot draw current. The digital voltmeter and cathode ray oscilloscope can measure accurate potential difference because they do not draw current from circuit because of their large resistance but these instruments are very expensive and are difficult to use. A very simple instrument which can measure and compare potential difference accurately is a potentiometer.

Uses:

- (i) Potential Divider
- (ii) To find unknown emf of a cell
- (iii) To compare emf of two cells

(i) Potential Divider:

Potentiometer consists of a resistor 'R' in form of a wire on which a terminal 'C' can slide Fig (a).

The resistance between 'A' and 'C' can be varied from 0 to R by moving 'C' from 'A' to 'B'. If a battery of emf E is connected across R fig (b) then current flowing through it is

$$I = \frac{E}{R}$$

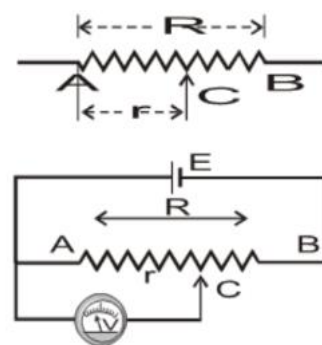
If resistance between A and C is r the potential difference b/w A and C is $V = I r$

$$V = \frac{E}{R} r$$

As C is moved from A to B, r varies from 0 to R and potential drop b/w A and C changes from 0 to E. Such an arrangement is known as a potential divider.

(ii) To find unknown emf of a cell:

It can be used to measure unknown emf E_x of a source by using the circuit shown in (Fig). Here R is in the form of straight wire of uniform area of cross section. A source of potential whose emf E_x is to



be measured is connected b/w A and sliding contact 'C' through a galvanometer. The positive terminal of E_x and positive terminal of potential divider are connected to the same point 'A'. If the point 'C' and negative terminal of E_x are at same potential, then terminals of galvanometer will be at same potential, and no current will flow through galvanometer. So, to measure E_x , the position of C is so adjusted that galvanometer shows no deflection. In this case Emf (E_x) of cell = potential difference b/w A and C

$$E_x = \frac{E}{R} r$$

$$E_x = E \frac{r}{R}$$

As resistance is proportional to length of wire

$$r \propto l ; R \propto L$$

$$E_x = E \frac{r}{R}$$

$$= E \frac{l}{L}$$

$$E_x = E \frac{l}{L}$$

Where

L = Total length of wire AB.

l = length of wire b/w A and C.

In equation $E_x = E \frac{l}{L}$, all values on right side are known so E_x can be calculated.

(iii) To compare EMF'S E_1 and E_2 of two cells:

The method for measuring emf of a cell can be used to compare emf E_1 and E_2 of two cells. The balancing E_1 and E_2 of two cells. The balancing lengths l_1 and l_2 are found separately for the two cells. Then,

$$E_1 = E \frac{l_1}{L} \text{ --- (1)}$$

$$E_2 = E \frac{l_2}{L} \text{ --- (2)}$$

Dividing eq. (1) by (2) we get

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Numerical Problems

9.1 Two unequal point charges repel each other with a force of 0.4 N when they are 5.0 cm apart. Find the force which each charge exerts on the other when they are a) 2.5 cm apart b) point 15.0 cm apart.

Given data:

$$F_1 = 0.4 \text{ N}$$

$$r_1 = 5.0 \text{ cm} = 0.05 \text{ m}$$

To find:

$$F_2 = ?$$

Solution:

$$F = k \frac{q_1 q_2}{r^2}$$

$$F_2 = F_1 \left(\frac{r_1}{r_2} \right)^2$$

(a) When

$$r_2 = 2.5 \text{ cm} = 0.025 \text{ m}$$

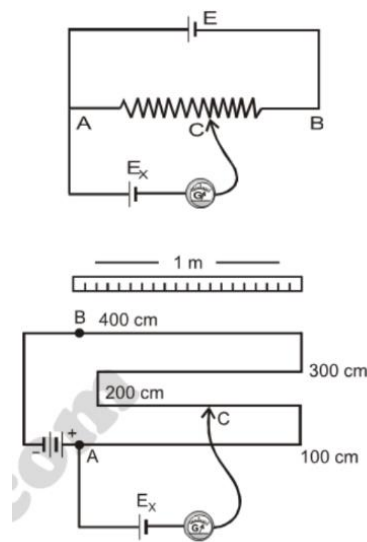
$$F_2 = 0.4 \left(\frac{0.05}{0.025} \right)^2$$

$$F_2 = 1.6 \text{ N}$$

Ans

(a) When

$$r_2 = 15.0 \text{ cm} = 0.15 \text{ m}$$



$$F_2 = 0.4 \left(\frac{0.05}{0.15} \right)^2$$

$$F_2 = 0.4 \times (0.33)^2$$

$$F_2 = 0.044 \text{ N} \quad \text{Ans}$$

9.2 A particle of charge $+20 \mu\text{C}$ is placed between two parallel plates, 10 cm apart and having a potential difference of 0.5 kV between them. Calculate the electric field between the plates and the electric force exerted on the charged particle.

Given:

$$q = +20 \mu\text{C} = +20 \times 10^{-6} \text{ C}$$

$$d = 10 \text{ cm} = 0.1 \text{ m}$$

$$V = 0.5 \text{ kV} = 500 \text{ V}$$

To find:

$$(a) \quad E = ?$$

$$(b) \quad F = ?$$

Solution:

$$(a) \quad E = \frac{V}{d}$$

$$E = \frac{500}{0.1}$$

$$E = 5000 \text{ V m}^{-1} = 5 \text{ kNC}^{-1} \quad \text{Ans}$$

$$(b) \quad F = q E$$

$$F = (20 \times 10^{-6}) \times (5000)$$

$$F = 0.1 \text{ N}$$

$$= 1000 \times 0.1 \text{ mN}$$

$$F = 100 \text{ mN} \quad \text{Ans}$$

9.3 The electron and proton in a hydrogen atom are separated (on the average) by approximately $5.3 \times 10^{-11} \text{ m}$. Find the ratio of the electric force and the gravitational force between the electron and proton in this state.

Given data:

$$r = 5.3 \times 10^{-11} \text{ m}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$\text{To find:} \quad \frac{F_e}{F_g} = ?$$

Solution:

$$F_e = k \frac{q_1 q_2}{r^2} \quad (\because q_1 = q_2 = e)$$

$$= k \frac{e^2}{r^2}$$

$$= (9 \times 10^9) \frac{(1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2}$$

$$F_e = 8.19 \times 10^{-8} \text{ N}$$

$$\text{As,} \quad F_g = G \frac{m_e m_p}{r^2}$$

$$= (6.63 \times 10^{-11}) \frac{9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}{(5.3 \times 10^{-11})^2}$$

$$F_g = 3.61 \times 10^{-47} \text{ N}$$

$$\frac{F_e}{F_g} = \frac{8.19 \times 10^{-8}}{3.61 \times 10^{-47}}$$

$$\frac{F_e}{F_g} = 2.3 \times 10^{39} \quad \text{Ans}$$

9.4 After a pleasant showering, a water droplet of mass $1.2 \times 10^{-11} \text{ kg}$ is located in the air near the ground and atmospheric electric field of magnitude $6.0 \times 10^3 \text{ NC}^{-1}$ points vertically

downward in the vicinity of the water droplet. The droplet remains suspended at rest in the air. Find the electric charge on the droplet.

Given:

$$m = 1.2 \times 10^{-11} \text{ kg}$$

$$E = 6.0 \times 10^3 \text{ NC}^{-1}$$

$$g = 9.8 \text{ ms}^{-2}$$

To find:

$$q = ?$$

Solution:

$$F_e = F_g$$

$$qE = mg$$

$$q = \frac{mg}{E}$$

$$q = \frac{1.2 \times 10^{-11} \times 9.8}{6.0 \times 10^3}$$

$$q = 1.96 \times 10^{-14} \text{ C} \quad \text{Ans}$$

9.5 An electron enters the region of a uniform electric field, with $v_i = 2.99 \times 10^6 \text{ m s}^{-1}$ and $E = 300 \text{ N C}^{-1}$. The horizontal length of the plates is 10.0 cm. Find the acceleration of the electron while it is in the electric field. How long will it take to pass through the field?

Given data:

$$v = 2.99 \times 10^6 \text{ m s}^{-1}$$

$$E = 300 \text{ N C}^{-1}$$

$$S = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$

To Find

$$a) \quad a = ?$$

$$b) \quad t = ?$$

Solution:

$$a) \quad \text{As, } F = ma \quad \& \quad F = qE$$

Comparing both equations,

$$ma = qE$$

$$a = \frac{qE}{m}$$

$$a = \frac{1.6 \times 10^{-19} \times 300}{9.11 \times 10^{-31}}$$

$$a = 5.27 \times 10^{13} \text{ m s}^{-2} \quad \text{Ans}$$

$$b) \quad S = vt$$

$$t = \frac{S}{v}$$

$$t = \frac{10 \times 10^{-2}}{2.99 \times 10^6}$$

$$t = 3.34 \times 10^{-8} \text{ s} \quad \text{Ans}$$

9.6 A disc of 10 cm^2 area is placed in a vertical electric field $E = 5 \times 10^5 \text{ N C}^{-1}$. If the plane of disc makes an angle of 30° with the horizontal, determine the electric flux through the disc.

Given data:

$$A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$$

$$E = 5 \times 10^5 \text{ N C}^{-1}$$

$$\theta = 30^\circ$$

To find:

$$\Phi = ?$$

Solution:

$$\Phi = EA \cos \theta$$

$$\Phi = 5 \times 10^5 \times 10 \times 10^{-4} \times \cos 30^\circ$$

$$\Phi = 250 \sqrt{3} \text{ N C}^{-1} \text{ m}^2 \quad \text{Ans}$$

9.7 A circular copper rod is 50 cm long and has 1 cm diameter. Find the resistance across its ends. What should be the side of square cross-section of a 50 cm long tungsten rod if its resistance is the same? [Resistivity of copper is $1.69 \times 10^{-8} \Omega \text{ m}$ and that of tungsten is $5.0 \times 10^{-8} \Omega \text{ m}$].

Given data:

$$L = 50 \text{ cm} = 0.5 \text{ m}$$

$$d = 1 \text{ cm} = 0.01 \text{ m}$$

$$\rho_{\text{Cu}} = 1.69 \times 10^{-8} \Omega \text{ m}$$

$$\rho_{\text{T}} = 5.0 \times 10^{-8} \Omega \text{ m}$$

To Find:

$$R_{\text{Cu}} = ?$$

$$\text{Side length} = L = ?$$

Solution: We know that

$$R = \rho \frac{L}{A}$$

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$$

$$= (3.14) \left(\frac{1 \times 10^{-2}}{2}\right)^2$$

$$A = 7.85 \times 10^{-5} \text{ m}^2$$

$$\text{Then, } R_{\text{Cu}} = \rho \frac{L}{A}$$

$$= 1.69 \times 10^{-8} \times \frac{0.5}{7.85 \times 10^{-5}}$$

$$R_{\text{Cu}} = 1.08 \times 10^{-4} \Omega \quad \text{Ans}$$

If the tungsten has same resistance as copper

$$R_{\text{T}} = \rho \frac{L}{A}$$

$$\text{or } A = \rho \frac{L}{R_{\text{Cu}}}$$

$$= (5 \times 10^{-8}) \times \frac{0.5}{1.08 \times 10^{-4}}$$

$$A = 2.3 \times 10^{-4} \text{ m}^2$$

For square bar

$$A = \text{length} \times \text{width}$$

$$A = L^2$$

$$\text{or } L = \sqrt{A}$$

$$= \sqrt{2.3 \times 10^{-4}}$$

$$= 0.0152 \text{ m}$$

$$L = 1.52 \text{ cm} \quad \text{Ans}$$

9.8 The copper winding of an electric fan has a resistance of at 50Ω at 30°C . After running for some time, the resistance becomes 52Ω . How much is the increase in temperature of the winding? [For copper $\alpha = 0.0039 \text{ K}^{-1}$]

Given:

$$R_0 = 50 \Omega$$

$$R_t = 52 \Omega$$

$$\alpha = 0.0039 \text{ K}^{-1}$$

$$t_1 = 30^\circ\text{C} = 293 \text{ K}$$

To Find:

$$\Delta t = ?$$

Solution:

$$\alpha = \frac{R_t - R_0}{R_0 \Delta t}$$

$$\text{or } \Delta t = \frac{R_t - R_0}{R_0 \alpha}$$

$$\Delta t = \frac{52 - 50}{50 \times 0.0039}$$

$$\Delta t = 10.3 \text{ K} = 10.3^\circ\text{C} \quad \text{Ans}$$

9.9 During an experiment a copper wire of 50 m long and 150 μm thick is hung vertically. Then a current of 1A is passed across its ends for 50 s. Find the resistance of the wire and the heat dissipated during this process. [Resistivity of copper is $1.69 \times 10^{-8} \Omega \text{ m}$.]

Given:

$$L = 50 \text{ m}$$

$$d = 150 \mu\text{m} = 150 \times 10^{-6} \text{ m}$$

$$I = 1 \text{ A}$$

$$t = 50 \text{ s}$$

$$\rho_{\text{Cu}} = 1.69 \times 10^{-8} \Omega \text{ m}$$

To Find:

a) $R = ?$

b) $H = ?$

Solution

a) $A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$

$$= (3.14) \left(\frac{150 \times 10^{-6}}{2}\right)^2$$

$$A = 1.767 \times 10^{-8} \text{ m}^2$$

The resistance of a wire is given by the formula:

$$R = \rho \frac{L}{A}$$

$$= 1.69 \times 10^{-8} \times \frac{50}{1.767 \times 10^{-8}}$$

$$R = 47.8 \Omega$$

Ans

$$H = I^2 R t$$

$$= (1)^2 \times (47.8) \times (50)$$

$$H = 2390 \text{ J}$$

Ans

9.10 The emf of a battery is 12 V. It is connected to a 3.6 Ω resistor. If the internal resistance of the battery is 0.2 Ω , what will be the terminal voltage across the battery?

Given:

$$E = 12 \text{ V}$$

$$R = 3.6 \Omega$$

$$r = 0.2 \Omega$$

To Find:

$$V_t = ?$$

Solution:

$$I = \frac{E}{R+r} = \frac{12}{(3.6+0.2)}$$

$$I = 3.16 \text{ A}$$

$$\text{So, } V = E - Ir$$

$$= 12 - (3.1579 \times 0.2)$$

$$= 12 - 0.6316$$

$$= 11.368 \text{ V}$$

$$V = 11.4 \text{ V}$$

Ans

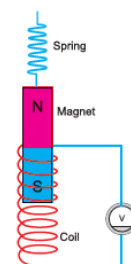
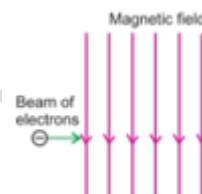
Chapter 10

Exercise

Electromagnetism

MCQs

- 10.1** A current is flowing towards north along a power line. The direction of the magnetic field over the wire is directed towards:
 (a) north (b) south (c) east (d) west
- 10.2** The radius of curvature of the path of a charged particle in a uniform magnetic field is directly proportional to:
 (a) the particle's charge (b) the particle's momentum
 (c) the particle's energy (d) the flux density of the field
- 10.3** The diagram shows a beam of electrons entering a magnetic field. What is the effect of magnetic field on the electrons?
 (a) They are deflected into the plane of the diagram.
 (b) They are deflected out of the plane of the diagram.
 (c) They are deflected towards the bottom of the diagram.
 (d) They are deflected towards the top of the diagram.
- 10.4** The force exerted on a wire of 1 meter length carrying 1 ampere current placed at right angle to the magnetic field is called:
 (a) magnetic field intensity (b) magnetic flux
 (c) magnetic induction (d) none of these
- 10.5** The unit of flux density is:
 (a) $\text{N A}^{-1} \text{m}^{-1}$ (b) N A m^{-1} (c) N m A^{-2} (d) N m A
- 10.6** A moving charged particle is surrounded by:
 (a) electric field only (b) magnetic field only
 (c) both electric and magnetic field (d) no field
- 10.7** Magnetic force on the charge q moving parallel to magnetic field with velocity v is:
 (a) $q v B \sin\theta$ (b) $q v B$ (c) zero (d) $I L B$
- 10.8** The unit $\text{N A}^{-1} \text{m}^{-1}$ is called:
 (a) weber (b) tesla (c) coulomb (d) none of these
- 10.9** Electrons while moving perpendicularly through a uniform magnetic field are:
 (a) deflected towards north pole (b) deflected towards south pole
 (c) deflected along circular path (d) not deflected at all
- 10.10** A magnet is suspended from a spring. The magnet oscillates and moves in and out of the coil connected to a galvanometer. When the magnet oscillates, the galvanometer shows:
 (a) deflection to the left and to the right with constant amplitude
 (b) deflected on one side
 (c) no deflection
 (d) deflection to the left and right, but the amplitude steadily decreases



Answer Key					
Q.No.	Option	Q.No.	Option	Q.No.	Option
10.1	C	10.5	A	10.9	C
10.2	B	10.6	C	10.10	D
10.3	B	10.7	C		
10.4	C	10.8	B		

Short Answers Question

10.1. It is said that Lenz's law specifically applies to induced currents and not directly to induced emf. Explain it briefly.

Ans Yes, Lenz's law specifically applies to the induced currents and not directly to induced emf.

This means we can apply Lenz's law to closed conducting loops or coils to determine the direction of induced current.

10.2 A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to magnetic field. Is an emf induced in the loop? Give a reason for your answer.

Ans No, emf will not be induced in the loop.

From the relation

$$\varepsilon = -N \frac{\Delta\phi}{\Delta t}$$

As magnetic field is uniform, therefore change of magnetic flux will be zero i.e. $\Delta\phi = 0$.

Hence $\varepsilon = 0$

10.3 Does the induced emf always act to decrease the magnetic flux through a circuit?

Ans No, induced emf does not always act to decrease the magnetic flux through a circuit. According to Lenz's Law, induced emf acts to maintain the magnetic flux by opposing any change in it. Thus, if magnetic flux decreases, it tends to increase the magnetic flux and if magnetic flux increases, it tends to decrease it.

10.4 When a magnet is pushed into the solenoid, as shown in the figure (a), the galvanometer indicates a small current. Why is the current produced? What will be the magnetic pole produced at the left end of the solenoid?

Ans According to the Faraday's law, a moving magnet changes the magnetic flux through the solenoid which causes induced emf. This induced emf produces an induced current which is indicated by the galvanometer.

According to the Lenz's law, direction of induced current always opposes the motion of the magnet so the face of the coil which is towards the magnet will be a north pole.

10.5 A bar magnet falls through a fixed metal ring (Fig-b). Will the magnet fall with an acceleration of a freely falling body? Give reason.

Ans No, it will fall with an acceleration less than gravitational acceleration (g).

When the magnet falls through the metal ring, it induces a current in the ring due to changing magnetic field. According to Lenz's law, the induced current opposes the magnet's motion, creating an upward force. This reduces its acceleration as compared to g.

10.6 Which of the two charged particles of the same mass will be deflected most in the magnetic field (a) fast moving (b) slow moving?

Ans The slow moving charged particle will be deflected most in the magnetic field.

When a charged particle enters a magnetic field, the magnetic force supplies the necessary centripetal force to move it in a circular path. Therefore,

$$F_B = F_C$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$r \propto v$ where m, q, B are constants

It is clear from the above relation that less is the speed, smaller will be the radius which means more deflection.

10.7 An electron and a proton are projected into a magnetic field at right angles to it with a certain velocity. Which of the particles will suffer greater deflection? Why?

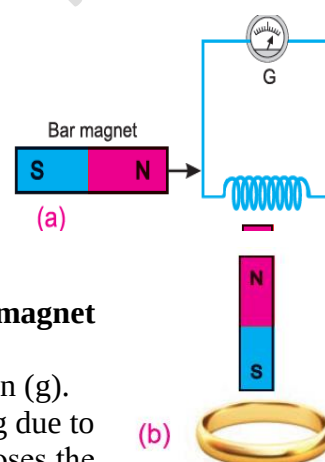
Ans The electron will suffer greater deflection than proton.

When a charged particle enters a magnetic field, the magnetic force supplies the necessary centripetal force to move it in a circular path. Therefore,

$$F_B = F_C$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$



As $r \propto m$ where v, q, B are constants
 $m_e < m_p$
 So, $r_e < r_p$

Therefore, electron will follow a smaller radius and thus its deflection will be greater than proton.

10.8 Can a single moving proton produce magnetic field?

Ans Yes, a single moving proton can produce a magnetic field.

According to principles of electromagnetism, all moving charged particles produce a magnetic field. As proton is a charged particle so a single moving proton produces magnetic field.

10.9 A magnetic field is necessary if there is to be a magnetic flux passing through a coil of wire. Yet, just because there is a magnetic field does not mean that a magnetic flux will pass through a coil. Account for this situation.

Ans A magnetic field is necessary for magnetic flux through a coil.

According to formula

$$\phi = BA \cos \theta$$

Magnetic flux depends on the angle between the field and normal to the coil.

If the magnetic field is parallel to coil ($\theta = 90^\circ$), then no magnetic flux passes through it.

Constructed Response Questions

10.1 A charge is lying stationary between the opposite poles of two magnets. Is a magnetic force exerted on it? Why?

Ans No, a magnetic force is not exerted on a stationary charge.

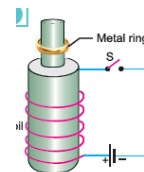
A magnetic force acts on a charge only when it moves in a magnetic field.

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

Since the charge is stationary, its velocity $\mathbf{v} = 0$, hence magnetic force on it is zero i.e., $\mathbf{F} = 0$.

10.2 When the switch in the circuit is closed, a current is established in the coil and the metal ring jumps upward. Why? Describe what would happen to the ring if the battery polarity were reversed?

Ans When switch is closed then current will increase from zero to maximum in the coil. This will induce the emf in the metallic ring. Due to induced emf, the magnetic field will be produced around the ring. This magnetic field will induce the current in the coil, which will oppose the magnetic field according to Lenz's law. Due to this opposition the ring jumps in the upward direction. In this case the S-poles of both the magnetic fields will repel each other. The ring will jump upward in the same manner, if the battery polarity is reversed. In this case the N-poles of both the magnetic fields will repel each other.



10.3 The figure shows a coil of wire in the x-y plane with a magnetic field directed along the y-axis. Around which of the three coordinate axes should the coil be rotated in order to generate an emf and a current in the coil?

Ans If the coil is rotated around x-axis, then emf and a current is generated in the coil due to change of magnetic flux passing through the coil.

10.4 Is it possible to change both the area of the loop and the magnetic field passing through the loop and still not have an induced emf in the loop? Give reason.

Ans Yes, it is possible.

$$\text{From the equation, } \Delta \phi = \Delta \mathbf{B} \cdot \Delta \mathbf{A}$$

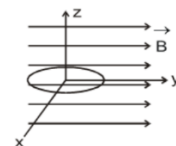
If we increase the area and equally decrease magnetic field and vice versa so that their product remains constant. Then, $\Delta \phi = 0$, hence there will be no induced emf.

10.5 Does the application of uniform magnetic field to a moving charged particle result in a change in kinetic energy of the particle? Explain.

Ans There will be no change in kinetic energy.

The magnetic force is always perpendicular ($\theta = 90^\circ$) to the velocity of moving charged particle and is given by the relation:

$$F_B = qvB \sin \theta$$



Since the work done by the magnetic force is zero, hence the particle's speed and kinetic energy remains the same.

10.6 A uniform electric field and a magnetic field act in the same direction. A proton is projected, into the space, with a uniform velocity in opposite direction. What will happen to the proton?

Ans The proton will decelerate due to opposite electric force applied by the electric field.

The relations for electric and magnetic forces are given as:

$$\mathbf{F}_e = q \mathbf{E}$$

$$\mathbf{F}_B = q (\mathbf{v} \times \mathbf{B}) = qvB \sin 180^\circ = 0$$

Hence, proton will move only under the influence of electric force.

10.7 A conductor moves in a magnetic field when a current is passed through the conductor. Would you expect the reverse effect to occur? That is, would a current be produced if a conductor is moved across the magnetic field?

Ans Yes, the reverse effect does occur.

According to Faraday's Law of electromagnetic induction, when a conductor moves across the magnetic field, it induces an emf. This emf drives a current in the conductor.

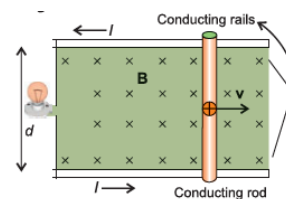
10.8 Consider a conducting rod of length L moving with velocity v to the right as shown in the figure. Left ends of the conducting rails are connected to a bulb. Due to motion of the rod through the magnetic field, an emf is produced across the ends of the rod. This emf gives rise to a current I . As a result, the bulb lights up. Explain where does the electrical energy consumed by the bulb come from?

Ans The mechanical energy is converted into electrical energy, which lights up the bulb.

The electrical energy consumed by the bulb is provided by the mechanical work done in moving the rod. As the rod moves through the magnetic field, it induces an emf, causing a current that lights up the bulb.

10.9 What will you do if you want to save a sensitive instrument from stray magnetic fields?

Ans To protect a sensitive instrument from stray magnetic fields, it is placed inside an iron box. Iron has high magnetic permeability, so it redirects the magnetic field lines and prevents them from affecting the instrument.



Comprehensive Questions

10.1 Distinguish between magnetic flux and flux density. How are they related?

Ans **Magnetic Flux ϕ_B :**

The magnetic flux through a patch of area A is the number of magnetic lines passing through this area.

The magnetic flux is given by the dot product of magnetic field $\vec{B}$ and vector area $\vec{A}$ is called magnetic flux.

$$\phi_B = \vec{B} \cdot \vec{A}$$

$$\phi_B = BA \cos \theta$$

The direction of $\vec{A}$ is along normal to plane surface element. Where θ is angle between vector $\vec{B}$ and $\vec{A}$.

Maximum Flux:

When area is held perpendicular to field $\vec{B}$ then $\theta = 0^\circ$ and flux is maximum.

$$\phi_B = \vec{B} \cdot \vec{A}$$

$$= BA \cos \theta$$

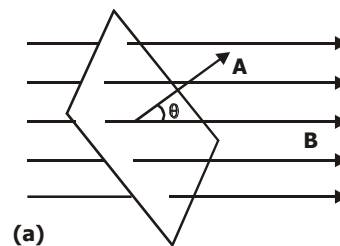
$$= BA \cos 0^\circ$$

$$\phi_B = BA \text{ (Max)}$$

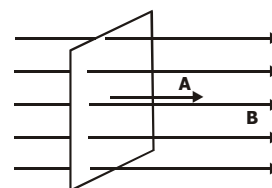
Minimum Flux:

When area is held parallel to field $\vec{B}$ then $\theta = 90^\circ$ and flux is minimum.

$$\phi_B = \vec{B} \cdot \vec{A}$$



(a)



(b)

$$\begin{aligned}
 &= BA \cos \theta \\
 &= BA \cos 90^\circ \\
 &= BA (0)
 \end{aligned}$$

$$\phi_B = 0 \text{ (Min)}$$

Unit of Magnetic Flux:

Unit of magnetic flux ϕ_B is weber (Wb).

$$\phi_B = \vec{B} \cdot \vec{A}$$

$$1 \text{ Wb} = \text{NA}^{-1} \text{m}^{-1} \cdot \text{m}^2$$

$$1 \text{ Wb} = \text{NA}^{-1} \text{m} = \frac{\text{Nm}}{\text{A}}$$

Magnetic Flux Density B:

Magnetic field strength (magnetic induction) B is also called magnetic flux density.

It is defined as the magnetic flux per unit area placed perpendicular to the magnetic field.

Relation:

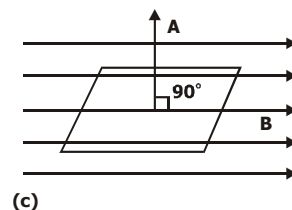
The relation between magnetic flux and flux density can be given as:

$$\text{Magnetic flux is } \phi_B = BA$$

$$\text{Magnetic flux density is } B = \frac{\phi_B}{A}$$

Unit of Magnetic Flux Density:

$$1 \text{ T} = \frac{1 \text{ Wb}}{1 \text{ m}^2} = \text{Wbm}^{-2} = \text{N A}^{-1} \text{m}^{-1}$$



10.2 Find an expression for the force exerted on a current carrying conductor placed in a uniform magnetic field.

Ans Explanation:

It has been observed experimentally that a current-carrying conductor placed in a magnetic field experiences a force. Consider a straight conductor carrying a steady current placed perpendicular to uniform magnetic field. Assume the direction of the current is out of the paper as shown.

A current-carrying conductor experiences a force due to the interaction between its own magnetic field and the external magnetic field. The magnitude of the force depends upon following factors.

(i) The force F is directly proportional to the current flowing through the conductor.

$$F \propto I$$

(ii) Force F is directly proportional to the length L of the conductor inside the magnetic field.

$$F \propto L$$

(iii) Force F is directly proportional to the strength of the applied magnetic field.

$$F \propto B$$

Combining all these factors,

$$F \propto ILB$$

$$F = kILB$$

In SI- units, the constant of proportionality $k = 1$,

$$F = ILB \text{ --- (1)}$$

Magnetic Strength:

The magnetic strength is equal to the force exerted on a conductor of length one metre carrying a current of one ampere, placed perpendicular to the magnetic field.

$$\text{From equation (1), } B = \frac{F}{IL}$$

Here B is the strength of magnetic field which is also known as magnetic induction.

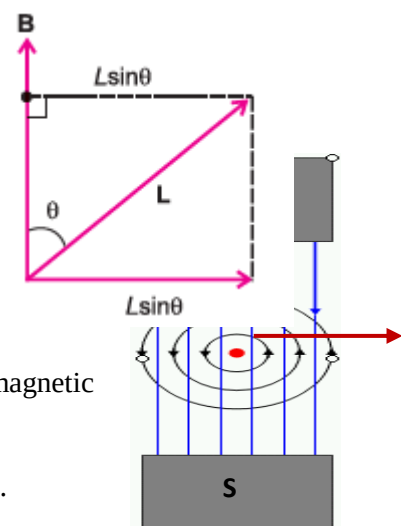
SI units:

The SI units of B is tesla.

$$1 \text{ T} = 1 \text{ N m}^{-1} \text{A}^{-1}$$

Vector Form:

Now consider conductor L placed at an angle 'θ' w.r.t the magnetic field, then we will use the component of L perpendicular to B i.e.,



$$F = BIL \sin \theta$$

In the vector form it can be written as

$$\mathbf{F} = I (\mathbf{L} \times \mathbf{B})$$

Maximum Force:

It shows that the force will be maximum (BIL) when the conductor is perpendicular to the field, i.e., $\theta = 90^\circ$,

$$F = BIL \sin 90^\circ = BIL$$

Minimum Force:

The force will be zero when the conductor is along the field i.e., $\theta = 0$.

$$F = BIL \sin 0^\circ = 0$$

Direction of Magnetic Force:

The direction of $\vec{F}$ can be determined by Fleming's Left Hand Rule:

Position your left hand such that the first finger points in the direction of magnetic field, the second finger points in the direction of current, thumb points in the direction of force.

10.2(A) Define motional emf and derive an expression for it.

Ans Motional EMF:

The emf induced by the motion of a conductor across a magnetic field is called motional emf.

$$\varepsilon = -vBL \sin \theta$$

Explanation:

Consider a conducting rod of length L placed on two parallel metal rails separated by a distance L . A galvanometer is connected between the ends c and d of the rails. This forms a complete conducting loop $abcd$ as shown in fig. A uniform magnetic field $\vec{B}$ is applied directed into the page. Initially when the rod is stationary, galvanometer shows no current in the loop. If the rod is pulled to the right with constant velocity $\vec{v}$, The galvanometer shows a current flowing through the loop.

The current is induced due to the motion of conducting rod across the magnetic field. The moving rod is acting as a source of emf ε .

$$\varepsilon = V_b - V_a = \Delta V$$

When the rod moves, a charge q also moves within the rod with the same velocity $\vec{v}$ in the magnetic field $\vec{B}$ and experiences a force $\vec{F}$ given by

$$\vec{F} = q (\vec{v} \times \vec{B})$$

The magnitude of the force $\vec{F}$ is

$$F = q v B \sin \theta$$

Since angle θ between $\vec{v}$ and $\vec{B}$ is 90° , so

$$F = qvB \sin 90^\circ$$

$$F = qvB \text{ ----- (1)}$$

Applying right hand rule, we see that $\vec{F}$ is directed from a to b in the rod. As a result, the charges migrate to the top end of the conductor. As more and more of the charges migrate, concentration of the charge is produced at the top b and deficiency of charges at the bottom a . This redistribution of charge sets up an electrostatic field E directed from b to a . The electrostatic force on the charge is

$$F_e = qE \text{ ----- (2)}$$

The system quickly reaches an equilibrium state in which these two forces on the charge are balanced.

If E_0 is the electric intensity in this state, then from eqns. (1) and (2),

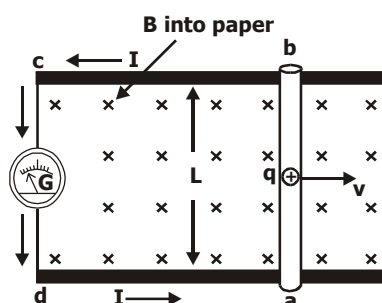
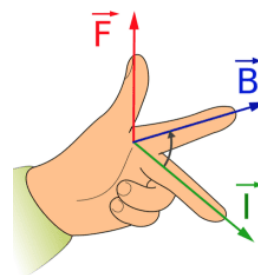
$$\begin{aligned} qE &= qvB \\ E &= vB \text{ ----- (3)} \end{aligned}$$

$$\text{As } E = -\frac{\Delta V}{L} \quad (\because E = -\frac{\Delta V}{\Delta r} = -\frac{\Delta V}{L})$$

$$E = -\frac{\varepsilon}{L} \text{ ----- (4) } (\because \varepsilon = \Delta V)$$

Comparing equation (3) and (4)

$$\frac{-\varepsilon}{L} = vB$$



The motional emf is

$$\varepsilon = -vBL$$

When the angle between $\vec{v}$ and $\vec{B}$ is θ , then

$$\varepsilon = -vBL \sin\theta$$

Taking the magnitude and ignoring the -ve sign

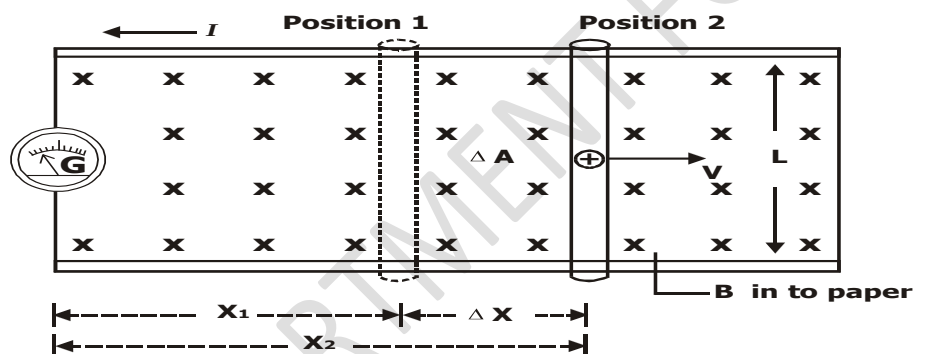
$$\varepsilon = vBL \sin\theta$$

Due to induced emf, positive charges would flow along the path abcd, so direction of current is anticlockwise as shown in the fig. As the current flows the quantity of charge at top decreases so the electric intensity decreases but the magnetic force remains the same. The current continues to flow due to the unbalanced magnetic field.

10.3 State and explain Faraday's law and Lenz's law. Also describe factors affecting the induced emf.

Ans Faraday's Law:

It is stated as: The average emf induced in a conducting coil of N loops is equal to the negative of the rate at which magnetic flux through the coil is changing with time.



$$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t}$$

Proof:

Let a conducting rod of length L moves from position 1 to 2 in a small interval of time Δt as shown in the fig. The distance travelled by the rod in time Δt is given by,

$$\Delta x = x_2 - x_1$$

Since the rod is moving with constant velocity v , therefore,

$$v = \frac{\Delta x}{\Delta t}$$

As

$$\varepsilon = -vBL$$

$$\varepsilon = -\frac{\Delta x}{\Delta t} BL$$

$$\varepsilon = \frac{-B(\Delta x.L)}{\Delta t} \text{-----(1)}$$

As the rod moves through the distance Δx , the increase in the area of the loop is given by

$$\Delta A = \Delta x L$$

This increases the flux through the loop by

$$\Delta\phi_B = B \Delta A$$

Putting value of ΔA ,

$$\Delta\phi_B = B.\Delta x.L$$

Putting this value in equation (1), we get

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t}$$

$$\text{For a coil of N loops, } \varepsilon = -N \frac{\Delta\phi_B}{\Delta t}$$

This is known as Faraday's Law of electromagnetic induction. The minus sign shows that direction of induced emf is such that it opposes the change in flux.

Factors affecting Emf:

1. Rated of Change of Magnetic Flux:

Faraday's law suggests that faster changes in magnetic flux result in greater induced emf.

2. Number of Turns of the Coil:

According to Faraday's law, induced emf is also proportional to the number of turns of the coil. More turns result in a greater induced emf.

3. Relative Speed:

The speed of the coil through a magnetic field also affects the magnitude of induced emf. Faster speed increases the rate of change of magnetic flux that results into increase in the induced emf.

10.4 Determine the force acting on a charged particle moving through a uniform magnetic field.

Ans When a charge 'q' moves in a magnetic field $\vec{B}$ with velocity $\vec{v}$ it experiences a force which is perpendicular to both $\vec{v}$ and $\vec{B}$ given as

$$\vec{F} = q(\vec{v} \times \vec{B})$$

We can calculate this force by examining the behavior of the conductor in magnetic field.

Consider a conductor of length L through which N charged particles, each carrying a charge q , pass in a time t . The motion of these charged particles generates a current I in the conductor, which is given by

$$I = \frac{Q}{t} = \frac{Nq}{t}$$

where Q is the total charge flowing in time t .

If $\mathbf{v}$ is the velocity of charged particles, then the velocity of the particles along the conductor is

$$\mathbf{v} = v\hat{L}$$

where $\hat{L}$ is the unit vector in the direction of the current.

The sign of the force depends on whether the charge q is positive or negative. However, the unit vector $\hat{L}$ is directed along the direction of the current, which is the direction of motion of positive charges.

Since the particles take time t to move across the conductor of length L , therefore,

$$t = \frac{L}{v}$$
$$I = \frac{Nq}{t} = \frac{Nqv}{L} \text{-----(1)}$$

If this conductor is placed in a uniform magnetic field B , the force is as given by

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$
$$\mathbf{L} = L\hat{L} \text{-----(2)}$$

As

Substituting the values of I and L , we get

$$\mathbf{F} = \frac{Nqv}{L}(L\hat{L} \times \mathbf{B})$$
$$\mathbf{F} = Nqv\hat{L} \times \mathbf{B}$$

As velocity $\mathbf{v}$ is directed along the conductor $\mathbf{L}$, so we can write

$$v\hat{L} = v\hat{v} = \mathbf{v}$$
$$\mathbf{F} = Nq\mathbf{v} \times \mathbf{B}$$

Therefore, the force on a single particle will be

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

If θ is the angle between $\mathbf{B}$ and $\mathbf{v}$, then magnitude of force is

$$F = qvB\sin\theta$$

Maximum magnetic force:

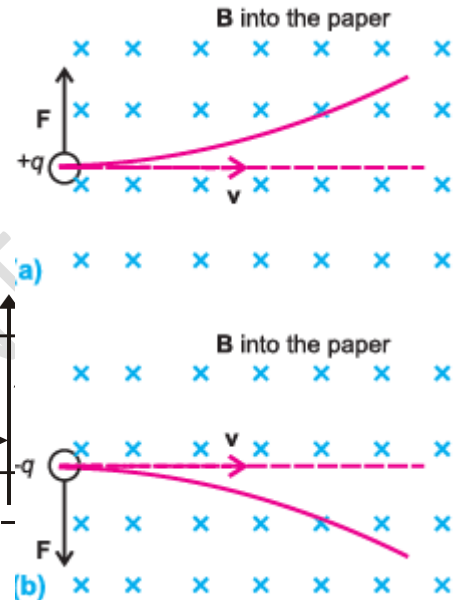
The force is maximum when $\mathbf{B}$ is perpendicular to $\mathbf{v}$ i.e.,

$$F = qvB\sin\theta$$
$$F_{\max} = qvB\sin 90^\circ = qvB$$

Minimum magnetic force:

The force is minimum when $\mathbf{B}$ is parallel to $\mathbf{v}$ i.e.,

$$F_{\min} = qvB\sin 0^\circ = 0$$



Direction of Force:

The direction of force can be known by applying Fleming's left-hand rule or right hand rule.

- The positively charged particle enters into the magnetic field along the dotted line on plane of paper. It experiences a force in the upward direction due to which it is deflected along a curved path (Fig. a).
- The negatively charged particle is deflected downward by the force acting on it downwards (Fig. b).

10.6 Explain how ferrofluids work?**Ans Ferrofluid:**

It is a unique material that exhibits both liquid and magnetic properties. It operates through a combination of magnetic and fluid dynamics principles.

Essentially, the ferrofluid is a colloidal suspension of magnetic particles in a carrier fluid (such as oil or water). Typically, the magnetic particles are iron oxide, ground to the nano-scale and approximately 10 nanometers in size. These particles are coated with a surfactant, a substance that reduces surface tension. This coating prevents the particles from clumping together, ensuring they remain evenly dispersed in the fluid. The viscosity of the fluid, the nanometer size of the particles, and their constant movement prevent the particles from settling down.

When there is no magnet around, a ferrofluid acts like a liquid, but when there is a magnet nearby, the particles are temporarily magnetized, and the fluid becomes a magnet. They form structures within the fluid causing the ferrofluid to act more like a solid. When the magnet is removed, the particles are demagnetized and the ferrofluid acts like a liquid again.

This phenomenon is due to the competition between magnetic forces, surface tension and gravity.

Spikes:

In the presence of strong magnetic field, the formation of chain-like structures is a result of magnetic forces pulling the fluid upwards while gravity and surface tension work to pull it back down. These chains align along the magnetic field lines and increase viscosity, making it behave like a solid bulging in certain directions. These are commonly known as spikes. However, the spikes are formed where the magnetic forces overcome the other forces.

Applications:

There are many applications of ferrofluids in the fields of electronics, medicine, engineering, and materials science.

In electronics, they are used in rotary seals for computer hard drives and loudspeakers.

In medicine, they can be used for targeted drug delivery and as contrast agents in MRI.

Numerical Problems

10.1 A positively charged particle is projected perpendicular into a magnetic field at a speed of 1500 m s^{-1} . It experiences a force of magnitude F . At what angle θ with the field, the particle should be projected at a speed of 2000 m s^{-1} , so that it experiences the same magnitude of force?

Given data:

$$\theta_1 = 90^\circ$$

$$v_1 = 1500 \text{ m s}^{-1}$$

$$v_2 = 2000 \text{ m s}^{-1}$$

To Find:

$$\theta_2 = ?$$

Solution:

$$F_1 = F_2$$

$$qv_1 B \sin\theta_1 = qv_2 B \sin\theta_2$$

$$v_1 \sin 90 = v_2 \sin\theta_2$$

$$1500 \times 1 = 2000 \times \sin\theta_2$$

$$\frac{1500}{2000} = \sin\theta_2$$

$$0.75 = \sin\theta_2$$

$$\theta_2 = \sin^{-1}(0.75)$$

$$\theta_2 = 49^\circ$$

Ans

10.2 Electrons are accelerated from rest through a potential difference of 15 kV in an oscilloscope. The electrons then pass through a 0.35 T magnetic field that deflects them to the desired position on the screen. Find the magnitude of the maximum force that an electron can experience.

Given data:

$$V = 15 \text{ kV} = 15 \times 10^3 \text{ V}$$

$$B = 0.35 \text{ T}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$q = e = 1.6 \times 10^{-19} \text{ C}$$

To Find: $F = ?$

Solution:

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m} = \frac{2 \times 1.6 \times 10^{-19} \times 15 \times 10^3}{9.11 \times 10^{-31}}$$

$$v = \sqrt{5.2 \times 10^{19}} = 7.2 \times 10^7 \text{ m s}^{-1}$$

As, $F_{\max} = evB \sin 90^\circ$

$$= 1.6 \times 10^{-19} \times 7.2 \times 10^7 \times 0.35$$

$$F_{\max} = 4.1 \times 10^{-12} \text{ N} \quad \text{Ans}$$

10.3 A square coil of side 15 cm each consists of 60 turns. Initially, it is located in a uniform magnetic field of magnitude 0.8 T such that plane of the coil is perpendicular to the field. The coil is then turned through an angle of $\theta = 30^\circ$ in a time of 2 s. Determine the average induced emf.

Given data:

$$L = 15 \text{ cm} = 0.15 \text{ m}$$

$$A = L \times L = (0.15)^2 = 0.0225 \text{ m}^2$$

$$N = 60$$

$$B = 0.8 \text{ T}$$

$$\theta_1 = 0^\circ$$

$$\theta_2 = 30^\circ$$

$$\Delta t = 2 \text{ s}$$

To Find: $\varepsilon = ?$

Solution:

$$\varepsilon = -N \frac{\Delta \phi}{\Delta t}$$

$$\varepsilon = -60 \frac{(\phi_2 - \phi_1)}{2} = -60 \frac{(BA \cos \theta_2 - BA \cos \theta_1)}{2}$$

$$= -30(0.8 \times (0.0225) \cos 30^\circ - 0.8 \times (0.0225) \cos 0^\circ)$$

$$= -30(0.0156 - 0.018)$$

$$= -30(-0.0024)$$

$$\varepsilon = 0.07 \text{ V} \quad \text{Ans}$$

10.4 A metallic rod is moving through a uniform magnetic field of 0.2 T. The emf induced across its ends is found to be 0.8 V. It is required to induce an emf of 2.4 V across its ends. How much field strength is needed for this?

Given data:

$$B_1 = 0.2 \text{ T}$$

$$\varepsilon_1 = 0.8 \text{ V}$$

$$\varepsilon_2 = 2.4 \text{ V}$$

To Find:

$$B_2 = ?$$

Solution:

$$\frac{B_2 v L \sin \theta}{B_1 v L \sin \theta} = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\frac{B_2}{B_1} = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\frac{B_2}{0.2} = \frac{2.4}{0.8}$$

$$\frac{B_2}{0.2} = 3$$

$$B_2 = 3 \times 0.2$$

$$B_2 = 0.6 \text{ T} \quad \text{Ans}$$

10.5 A copper ring has a radius of 4.0 cm and resistance of 1.0 mΩ. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2 T to 0.4 T in a time interval of 5×10^{-3} s, what is the current in the ring during this interval?

Given data:

$$N = 1$$

$$r = 4.0 \text{ cm} = 4 \times 10^{-2} \text{ m}$$

$$R = 1.0 \text{ m}\Omega = 1 \times 10^{-3} \Omega$$

$$\theta = 0^\circ$$

$$\Delta B = B_2 - B_1 = (0.4 - 0.2) = 0.2 \text{ T}$$

$$\Delta t = 5 \times 10^{-3} \text{ s}$$

To Find:

$$I = ?$$

Solution:

$$\begin{aligned} I &= \frac{\varepsilon}{R} = \frac{1}{R} \times \left(- \frac{N \Delta \phi}{\Delta t} \right) \\ &= - \frac{N}{R} \times \frac{\Delta B A}{\Delta t} = - \frac{N}{R} \times \frac{(\Delta B) \pi r^2}{\Delta t} \\ &= - \frac{1}{1 \times 10^{-3}} \times \frac{0.2 \times 3.14 (4 \times 10^{-2})^2}{5 \times 10^{-3}} \end{aligned}$$

$$I = 201 \text{ A} \quad \text{Ans}$$

10.6 A coil of 10 turns and 35 cm² area is in a perpendicular magnetic field of 0.5 T. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field.

Given data:

$$N = 10$$

$$A = 35 \text{ cm}^2 = 35 \times 10^{-4} \text{ m}^2$$

$$\Delta B = 0.5 \text{ T}$$

$$t = 1.0 \text{ s}$$

To find:

$$\varepsilon = ?$$

Solution:

$$\begin{aligned} \varepsilon &= N \frac{\Delta \phi}{\Delta t} = N \frac{\Delta B A}{\Delta t} \\ \varepsilon &= 10 \frac{(0.5 \times 35 \times 10^{-4})}{1.0} \end{aligned}$$

$$\varepsilon = 1.75 \times 10^{-2} \text{ V} \quad \text{Ans}$$

10.10 An emf of 0.5 V is induced across the ends of a metal rod moving through a magnetic field of 0.4 T. If an emf of 1.5 V has to be induced, what field strength would be needed for that? Assume that all other factors remain same.

Given data:

$$\varepsilon_1 = 0.5 \text{ V}$$

$$\varepsilon_2 = 1.5 \text{ V}$$

$$B_1 = 0.4 \text{ T}$$

$$B_2 = ?$$

To find:

Solution:

$$\begin{aligned} \frac{B_2}{B_1} &= \frac{\varepsilon_2}{\varepsilon_1} \\ B_2 &= \frac{\varepsilon_2}{\varepsilon_1} \times B_1 \\ &= \frac{1.5}{0.5} \times 0.4 \\ B_2 &= 1.2 \text{ T} \quad \text{Ans} \end{aligned}$$

Q. 1. What is relative motion? Explain with example.

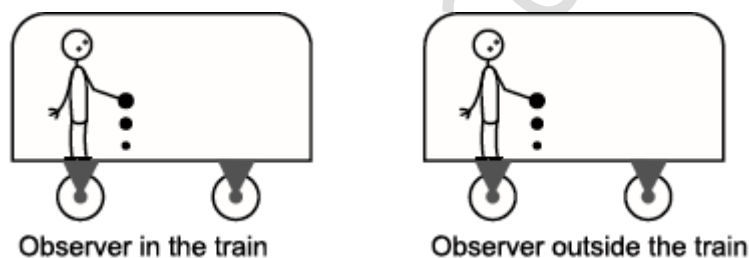
Ans. “The change of position of a body with respect to surrounding (other body) is called relative motion.”

Explanation: The state of rest or motion depends upon the observer. For example, the walls of moving train seem stationary to passengers inside the train but appear to be moving to someone standing on the ground. Therefore, we cannot definitively say whether an object is absolutely at rest or in motion because all motions are relative to an observer.

Example: An observer in a closed train compartment uses the compartment as his frame of reference. To determine the train’s motion, the observer drops a ball and measures the horizontal distance travelled by the wall, keeping the vertical distance same as ‘t’ seconds in all cases.

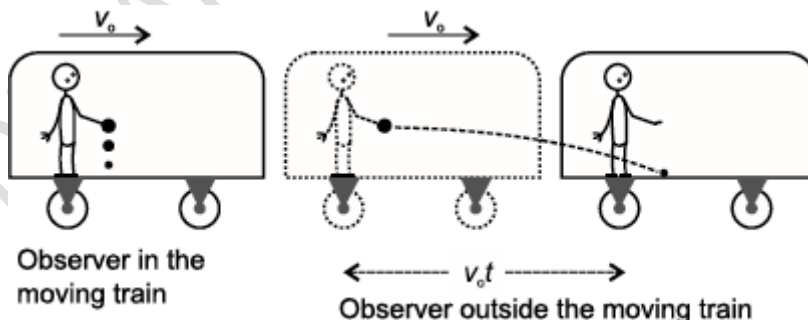
Case (a) When train is stationary

In this case the horizontal velocity and distance travelled will be zero. The observations made by the observer inside and outside of the train will be same. The ball will have fallen to a point on the floor directly below the point from where it was dropped as shown in fig.



Case (b) When train is moving with uniform velocity v_0

When ball is released, it has initial horizontal velocity v_0 and behaves like a projectile. It travels a horizontal distance $S = v_0 t$, t is time the ball takes to reach the floor. Since both the train and the observer inside it are moving with the same velocity v_0 , they both travel the same horizontal distance S in same time t . Therefore, the observer inside the train sees that the ball falls to a point on the floor directly below where it was dropped. In contrast, an observer outside the train will see the ball following a projectile path as shown in fig.



Conclusion: The observers in different frames of reference, moving with uniform velocity relative to each other, will perceive motion differently.

Q.2. What is meant by the “frame of reference”? Distinguish between an inertial frame of reference and a non-inertial frame of reference by giving examples.

Ans. Frame of Reference:

A frame of reference is any coordinate system relative to which measurements are taken.

Examples:

The position of a table in a room can be described relative to the walls of the room, making the room the frame of reference. Similarly, the laboratory is the reference frame for measurements taken there. If the same experiment is performed in a moving train, the train becomes the frame of reference. The position of a spaceship can be described relative to the positions of distant stars, making a coordinate system based on these stars from the frame of reference.

Inertial Frame of Reference:

The frame that is at rest or moving with uniform velocity ($a = 0$) is called an inertial frame of reference or a non-accelerated frame of reference.

For such a frame, the law of inertia must hold. Other laws of nature also apply in such a system. According to the law of inertia, a body remains at rest or moving with uniform velocity unless an unbalanced force produces acceleration in it. Any frame of reference that is moving with uniform velocity relative to an inertial frame is also inertial.

Examples:

- Earth may be considered as an inertial frame of reference.
- A body placed in a car moving with uniform velocity with respect to the earth also remains at rest; that car is also an inertial frame of reference.

Non-Inertial Frame of Reference:

An accelerated frame of reference is called a non-inertial or accelerated frame of reference.

Examples:

- When a car is moving, and suddenly the brakes are applied. Then the body placed in it no longer remains at rest. So, due to the acceleration car is an accelerated frame of reference.
- Earth is rotating and revolving, and hence, strictly speaking, the Earth is not an inertial frame. But it can often be treated as an inertial frame without serious error because of a very small acceleration.

Q.3 What is the special theory of relativity? Write its postulates also discuss its results.

Ans: Special Theory of Relativity

The special theory of relativity deals with problems involving non-accelerated frames of reference.

It has two postulates, which can be stated as:

The laws of physics are the same in all inertial frames of reference.

The speed of light in free space has the same value for all observers regardless of their state of motion.

Results of Special Theory of Relativity:

1. The Relativity of Simultaneity:

If two events in different locations are observed by one observer to be simultaneous, they will generally not be observed as simultaneous by another observer in a different frame of reference moving relative to the first observer.

Consider a train equipped with light operated doors. The light switch is located in the centre of the roof and is operated by a traveler standing in the middle of the compartment. When the train is travelling at half the speed of light, the traveler turns on the light. The light travels forward and backward at equal speed and reaches both doors at the same time. Hence the traveler sees both doors opening simultaneously. However, an observer outside the train will see the back door open before the front door. This is because the back door is moving towards the light waves, while the front door is moving away from the light waves.

2. Time Dilation:

According to special theory of relativity, time is not absolute quantity. It depends upon the motion of the frame of reference. Suppose an observer is stationary in an inertial frame. He measures the time interval between two events happening successively in this frame. Let this time interval be 't₀'. This is known as proper time. If the observer is moving with respect to the frame with velocity 'v' or the frame is moving with respect to the observer with a uniform velocity 'v', the time measured by the observer would not be 't₀', but it would be 't' such that.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As 'c' is the velocity of light is very greater than 'v', therefore, the term $\frac{v^2}{c^2}$ is very small. Thus,

the quantity $\sqrt{1 - \frac{v^2}{c^2}}$ is always less than one, so according to the result of first postulate of relativity the time 't' in the moving frame is greater than the time 't₀' in the stationary frame.

We conclude that time has dilated or stretched due to relative motion of the observer and the frame of reference of events.

3. Length Contraction

If you are in motion relative to two points that are at fixed distance apart, the distance between the two points appears shorter if you were at rest relative to them. This effect is known as length contraction. The length of an object or distance between two points measured by an observer at rest is denoted by ' l_0 '. If an object and an observer are in relative motion with speed ' v ', then the contracted length ' l ' is given by

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

As ' v ' is less than ' c ', thus $\sqrt{1 - \frac{v^2}{c^2}} < 1$, i.e. $l < l_0$. It means that, the length measured in the moving spaceship would be smaller than it is measured at rest on the Earth.

4. Mass Variation

According to a special theory of relativity, mass of an object is a varying quantity and depends upon the speed of the object. If mass of an object at rest is ' m_0 ' and its increased mass moving at speed ' v ' is ' m ', then

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As $\sqrt{1 - \frac{v^2}{c^2}} < 1$ the relativistic mass ' m ', the mass of the body in motion at the speed ' v ', is always greater than ' m_0 ', the mass of the body when it is at rest.

As ' v ' approaches ' c ', it requires a large force to change the speed of the object.

$$\text{As } v \rightarrow c, \quad \frac{v}{c} = 1 \quad \text{therefore} \quad \sqrt{1 - \frac{v^2}{c^2}} \rightarrow 0$$

$$\text{Thus } m = \frac{m_0}{0} \text{ or } m \rightarrow \infty$$

An infinity mass would require an infinity force to accelerate it. Because infinity forces are not available, hence, an object cannot be accelerated to the speed of light ' c ' in free space.

5. Energy-Mass Relation

According to special theory of relativity, mass and energy are different quantities but are interchangeable. The total energy ' E ' and mass ' m ' of an object are related by the expression

$$E = mc^2 \dots\dots\dots(1)$$

It means that the product of mass times the square of the speed of light must equal to energy.

When a particle is at rest, its kinetic energy is equal to zero. Therefore, the total energy of the particle when it is at rest must be equal to m_0c^2 . The rest mass energy of a particle can then be defined as

$$E_0 = m_0c^2 \dots\dots\dots(2)$$

As mc^2 is greater than m_0c^2 , the difference of energy is due to motion, as such it represents the kinetic energy of the mass. Hence

$$\text{K.E.} = mc^2 - m_0c^2$$

$$\text{K.E.} = (m - m_0) c^2$$

From Eq. (1), change in mass ' m ' due to change in energy ΔE is given by

$$\Delta E = \Delta mc^2$$

$$\text{or} \quad \Delta m = \frac{\Delta E}{c^2}$$

As c^2 is very large quantity, this implies that small changes in mass require very large changes in energy.

Q.4 Describe the Einstein's mass-energy equation; why cannot we observe its effects in everyday life? What are its significance consequences? Give examples.

Ans: Energy-Mass Relation

According to special theory of relativity, mass and energy are different quantities but are interchangeable. The total energy 'E' and mass 'm' of an object are related by the expression

$$E = mc^2 \dots\dots\dots(1)$$

It means that the product of mass times the square of the speed of light must equal to energy. When a particle is at rest, its kinetic energy is equal to zero. Therefore, the total energy of the particle when it is at rest must be equal to m_0c^2 . The rest mass energy of a particle can then be defined as

$$E_0 = m_0c^2 \dots\dots\dots(2)$$

As mc^2 is greater than m_0c^2 , the difference of energy is due to motion, as such it represents the kinetic energy of the mass. Hence

$$\text{K.E.} = mc^2 - m_0c^2$$

$$\text{K.E.} = (m - m_0) c^2$$

From Eq. (1), change in mass 'm' due to change in energy ΔE is given by

$$\Delta E = \Delta mc^2$$

or
$$\Delta m = \frac{\Delta E}{c^2}$$

As c^2 is a very large quantity, this implies that small changes in mass require very large changes in energy. In our everyday world, energy changes are too small to provide measurable mass changes. However, energy and mass changes in nuclear reactions are found to be exactly in accordance with the aforementioned equations.

RELATIVE MOTION, FRAMES OF REFERENCE, SPECIAL THEORY OF RELATIVITY

1. All motions are:

- (a) Absolute (b) Uniform (c) Relative (d) Variable

2. Theory of relativity which deals with non-inertial frames of reference is called:

- (a) Special theory of relativity (b) General theory of relativity
(c) Quantum theory (d) Classical theory

3. Einstein presented special theory of relativity in:

- (a) 1850 (b) 1920 (c) 1905 (d) 1932

4. The rest mass of photon is:

- (a) m_0 (b) m_0c^2 (c) Infinity (d) Zero

5. If the speed of light were infinite, then moving mass $m =$ _____:

- (a) ∞ (b) 0 (c) $2m_0$ (d) m_0

6. The Einstein mass-energy equation is:

- (a) $E = mc$ (b) $E = mc^2$ (c) $E = mc^3$ (d) $E = m^2c^2$

7. Two photons approach each other; the relative speed will be:

- (a) $2c$ (b) zero (c) less than c (d) c

8. The coordinate system in which law of inertia is valid is called:

- (a) Special frame of reference (b) Inertial frame of reference
(c) Non-inertial frame of reference (d) Standard frame of reference

9. An inertial frame of reference is the one in which a body:

- (a) moves with uniform velocity (b) is at rest
(c) has zero acceleration (d) all of these

10. If a material object moves with speed of light, its mass becomes:

- (a) double (b) equal to its rest mass (c) four times (d) infinite

11. 1 kg mass will be equivalent to:

- (a) $9 \times 10^8 \text{ J}$ (b) $9 \times 10^{12} \text{ J}$ (c) $9 \times 10^{16} \text{ J}$ (d) $9 \times 10^{19} \text{ J}$

12. The length contraction happens only:

- (a) perpendicular to the direction of motion (b) along the direction of motion
(c) opposite to the direction of motion (d) along any direction

13. The observations on objects moving very fast, approaching the speed of light are well explained by:

- (a) Quantum Theory (b) Newton's theory (c) Kepler's Law (d) Special Theory of Relativity
- 14. The event on which all observers are agreed upon:**
 (a) speed of light (b) length measurement (c) time measurement (d) mass measurement
- 15. The length of an object becomes half of its proper length at the speed of:**
 (a) 0.5c (b) 0.866c (c) 0.707c (d) infinite
- 16. Earth's orbital speed is:**
 (a) 10 kms⁻¹ (b) 20 kms⁻¹ (c) 30 kms⁻¹ (d) 40 kms⁻¹
- 17. Which of the following do not travel at speed of light?**
 (a) Heat waves (b) Radio waves (c) X-rays (d) Sound waves
- 18. At what speed does the mass of an electron become double of its rest mass?**
 (a) 0.5c (b) 0.77c (c) 0.866c (d) 0.99c

Answer's Key

1	C	2	B	3	C	4	D	5	D	6	D	7	D	8	B	9	D	10	D
11	C	12	B	13	D	14	A	15	B	16	C	17	D	18	C				

Q.5. State the Einstein's concept about the space-time. Describe the view of gravity according to this concept.

Ans: Space-Time in Relativity

Space and Time:

Space is said to be a three-dimensional extent in which all objects and events occur. It provides a framework to define the position and motion of various objects under the influence of some force whereas time measures the sequence and duration of events.

In the theory of relativity, time is not absolute; it is considered the fourth dimension. For example, oscillatory motion, such as that of a swinging pendulum, relies on time to determine the frequency of oscillations.

Relationship between space and time:

The special theory of relativity explains that space and time are related to each other. It describes how space-time are influenced by gravity and speed, such as the bending of light around massive objects like stars. Space-time is, in fact, a mathematical model that unifies space-time into a single continuum. It is a concept used to describe all points of space and time and their relation to each other.

Newton's Theory of Gravitation:

According to Newton's theory, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

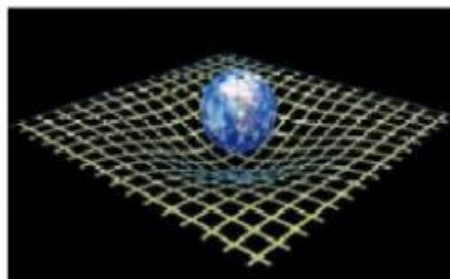
In other words, gravitation is due to the force between masses.

Newton discovered the inverse square law of gravity but explicitly stated that he offered no explanation for why gravity should follow this law.

Einstein's theory of gravitation:

Einstein's theory provides a physical picture of how gravity works. According to Einstein's theory, gravity is due to the curvature of space and time caused by masses.

To observe this, we think the space as a thin rubber sheet. If a heavy weight is hung from it, it curves as shown in figure.



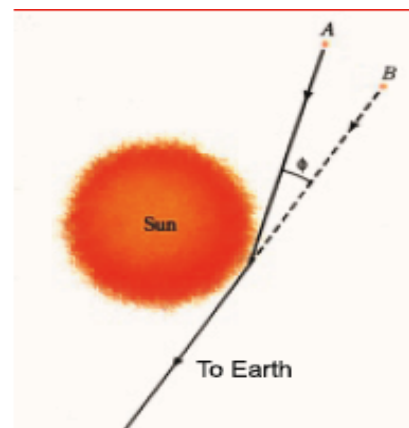
Einstein's theory also describes gravity as following an inverse square law (except in strong gravitational fields), but it explains why this is so. This is why Einstein's theory is considered an advancement over Newton's theory, even though it encompasses Newton's theory and yields the same results as Newton's theory in all but very strong gravitational situations.

The bending of starlight caused by the Sun's gravity was measured during a solar eclipse in 1919. The results matched Einstein's theory being hailed as a scientific triumph. Another success of Einstein's theory was the detection of gravitational waves, produced by some celestial events causing disturbances (squeezes and stretches) in the curvature of space-time. These waves were detected in 2015 and announced in 2016.

Hypothetical Examples of Space Time:

Let a spaceship be travelling to a star with half of the speed of light. Let it takes eight years to reach to the star, from the point of view of the observer on the Earth. From the Earth's point of view, the clocks on the spaceship are moving slowly, so that less time passes on the spaceship compared to the Earth.

For the spaceship occupants, the length of the journey has contracted which they cover in less time. The occupants of the spaceship record 7 years to reach their destination, rather than 8 years.



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ . Einstein predicted that $\phi = 1.745$ seconds of angle which was found to be the same during the solar eclipse of 1919.

SPACE-TIME IN RELATIVITY

- Relativistic mechanics yields result different from classical mechanics for objects moving with:**
 - low velocity
 - velocity equal to that of sound waves
 - velocity greater than sound waves
 - velocity approaching speed of light
- If an observer is moving in the same direction as sound wave, the velocity of the wave seems to be:**
 - more
 - less
 - constant
 - sum of the two velocities
- The speed of beam of light of a car while moving with high speed as compared to its rest position is:**
 - greater
 - less
 - same
 - zero
- Einstein's theory gives us a physical picture of how the:**
 - body moves
 - gravity works
 - moment of inertia is produced
 - weightlessness is created
- According to Einstein's theory, space time is:**
 - Linear
 - Curved
 - Circular
 - Elliptical
- According to Einstein space objects and light follow:**
 - Rectilinear path
 - Circular path
 - Geodesics
 - Elliptical path
- Who discovered the inverse square law of gravitational field?**
 - Einstein
 - Newton
 - Maxwell
 - Coulomb
- According to theory of relativity, the fourth dimension is:**
 - Time
 - Mass
 - Gravity
 - Sound
- The reason behind space time curvature is:**
 - Light rays
 - Massive body
 - Sound waves
 - Magnetic field
- The bending of light was observed during solar eclipse in:**
 - 1929
 - 1906
 - 1919
 - 2015

Answer's Key

1	D	2	C	3	C	4	B	5	B	6	C	7	B	8	A	9	B	10	C
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PHYSICS DEPARTMENT FCC

Exercise

MCQs

- 11.1** Relative mechanics yields results different from classical mechanics for objects moving with:
 (a) low velocity (b) velocity equal to that of sound waves
 (c) velocity greater than sound waves (d) velocity approaching that of light
- 11.2** If an observer is moving in the same direction as a sound wave, the velocity of the wave seems to be:
 (a) more (b) less (c) constant (d) sum of the two velocities
- 11.3** If the rest mass of a particle m_0 increases to m due to its high speed, then its kinetic energy is:
 (a) $\frac{1}{2} m c^2$ (b) $\frac{1}{2} m v^2$ (c) $(m - m_0) c^2$ (d) $\frac{1}{2} (m - m_0) c^2$
- 11.4** The speed of a beam of light of a car while moving with high speed as compared to its rest position is:
 (a) greater (b) less (c) same (d) zero
- 11.5** A photon is a particle of light. What is its mass when it moves with $0.9 c$?
 (a) $9.1 \times 10^{-31} \text{ kg}$ (b) $1.67 \times 10^{-19} \text{ kg}$ (c) $1.67 \times 10^{-27} \text{ kg}$ (d) Zero

Answer Key							
Q.No.	Option	Q.No.	Option	Q.No.	Option		
11.1	D	11.4	C				
11.2	B	11.5	D				
11.3	C						

Short Answer Questions

11.1 What is meant by inertial frame of reference and non-inertial frame of reference?

• **Inertial Frame:** An inertial frame of reference is one in which Newton's First Law holds true. In such a frame, a body either remains at rest or moves with uniform velocity unless acted upon by an unbalanced force.

Example: A body placed on the Earth remains at rest or moving with uniform velocity unless an unbalanced force acts on it.

• **Non-Inertial Frame:** A Non-inertial frame of reference is one in which Newton's First law (law of inertia) does not hold true.

Example: If a car is suddenly stopped or accelerated, it is a non-inertial reference frame.

11.2 What are the two postulates of the special theory of relativity?

1) Principle of Relativity: The laws of physics are the same in all inertial frames of reference.

2) Principle of Constancy of Light: The speed of light in free space has the same value for all observers, regardless of the state of motion of the source or the observer.

11.3 Describe why it is impossible for a material particle to move with speed of light.

A material particle cannot move with the speed of light as per special theory of relativity.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

As $v \rightarrow c$, the mass $m \rightarrow \infty$.

An infinite mass would require an infinite force to accelerate it. Since infinite forces are not available hence, an object cannot be accelerated to the speed of light c in free space.

11.4 Does theory of relativity contradict Newton's laws of motion? Explain briefly.

No, special theory of relativity does not contradict Newton's laws of motion.

Newton's laws are valid at low speeds. At speeds close to the speed of light, relativistic mechanics applies. It refers that the special theory of relativity extends Newton's laws of motion.

11.5 What is meant by proper time and proper length?

Proper Time (t_0): The time measured by a clock which is at rest relative to an event.

Proper Length (ℓ_0): The length of an object or the distance between two points measured by an observer who is at rest relative to them is called the proper length (ℓ_0).

11.6 What is meant by relativistic mass, length, and time?

Relativistic Mass:

According to special theory of relativity, when an object moves with the relativistic speed, its mass increases. This is called relativistic mass (m).

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Where m_0 is the rest mass and v is the velocity of an object.

Relativistic Length:

According to special theory of relativity, when an object moves with the relativistic speed, its length contracts. This is called relativistic length (ℓ). It occurs only along the direction of motion.

$$\ell = \ell_0 \sqrt{1 - v^2/c^2}$$

Relativistic Time:

According to special theory of relativity, when a clock moves with the relativistic speed, its time dilates. This is called relativistic time (t).

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

11.7 Why mass of a moving object increases?

According to the special theory of relativity, the mass of an object depends upon the object's speed. As the object's speed increases, more energy is required to accelerate it. This energy contributes to its relativistic mass, hence mass increases.

11.8 All motions are relative. Does space-time is absolute? Explain briefly.

Space and time are not absolute in special theory of relativity. They form a unified four-dimensional continuum called space-time. The measurement of time and length depends on the observer's frame of reference.

11.9 Explain that speed of light is an ultimate limit for any object.

According to special theory of relativity, no material object can be accelerated to or beyond the speed of light. As velocity approaches c , relativistic mass and energy requirements approach infinity, which is physically impossible.

11.10 Give examples where the results of special theory of relativity have been verified.

- 1) **Time Dilation in Muon Decay:** Muons created in the upper atmosphere reach Earth due to time dilation.
- 2) **Particle Accelerators:** Particles gain relativistic mass at high speeds as predicted by special theory of relativity.

Constructed Response Questions

11.1 Speed of sound is affected by relative motion between the observer and the source. Does this apply to speed of light as well? Describe briefly.

Ans No, the speed of light is not affected by the relative motion between the observer and the source. According to Einstein's special theory of relativity, the speed of light in a vacuum is constant (3×10^8 m/s Approx.) for all observers, regardless of their relative motion. This constancy is one of the fundamental postulates of special theory of relativity.

11.2 Is it ever possible to see a star moving away from us at a uniform velocity equal to the velocity of light?

Ans No, it is not possible.

According to the special theory of relativity, no material object can reach or exceed the speed of light. Therefore, a star cannot move away from us at a velocity equal to the speed of light. Only massless particles like photons can travel at that speed.

11.3 If the speed of light is just 50 ms^{-1} , how would everyday events appear to?

Ans If the speed of light were only 50 ms^{-1} , relativistic effects such as time dilation and length contraction would become noticeable even at everyday speeds. Events would appear distorted.

- Moving objects would visibly contract in length.
- Clocks on moving objects would tick much slower than those at rest.
- Communication and perception would be drastically delayed, making daily interactions extremely unnatural.

11.4 If the speed of light were infinite, what would the equations of special theory of relativity reduce to?

Ans If the speed of light were infinite, The equations of special theory of relativity reduce to

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{If } c = \infty \quad \text{then } m = \frac{m_0}{\sqrt{1 - \frac{v^2}{\infty^2}}} = \frac{m_0}{\sqrt{1 - 0}} = m_0$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{If } c = \infty \quad \text{then } t = \frac{t_0}{\sqrt{1 - \frac{v^2}{\infty^2}}} = \frac{t_0}{\sqrt{1 - 0}} = t_0$$

$$\ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2}} \quad \text{If } c = \infty \quad \text{then } \ell = \ell_0 \sqrt{1 - \frac{v^2}{\infty^2}} = \ell_0 \sqrt{1 - 0} = \ell_0$$

$$E = mc^2 \quad \text{If } c = \infty \quad \text{then } E = \infty$$

11.5 According to Einstein's equation; $E = mc^2$, is it possible to create a single electron from energy? Explain.

Ans No. It is not possible to create single electron from energy.

The creation of two particles with equal and opposite charges is necessary for charge conservation in the universe. Hence an electron and a positron (an anti-particle of electron) are created in pair production.

Numerical Problems

11.1 An electron is accelerated to a speed of $0.995c$ which passes down an evacuated tube 500 m long. How long will the tube appear to the electron?

Given Data:

$$v = 0.995c$$

$$\ell_0 = 500 \text{ m}$$

To find:

$$\ell = ?$$

Solution:

$$\begin{aligned} \ell &= \ell_0 \sqrt{1 - \frac{v^2}{c^2}} \\ &= 500 \times \sqrt{1 - \frac{(0.995c)^2}{c^2}} = 500 \times \sqrt{\frac{c^2(1 - (0.995)^2)}{c^2}} \\ \ell &= 500 \times \sqrt{1 - (0.995)^2} = 500 \times \sqrt{1 - 0.990025} \\ \ell &= 500 \times \sqrt{0.009975} \approx 500 \times 0.09987 \approx 49.94 \text{ m} \approx 50 \text{ m} \\ \ell &= 50 \text{ m} \end{aligned}$$

11.2 A neutron, being not a stable particle, disintegrates in 20 minutes on the average. How long will it seem to exist if it shoots out from a nucleus with a speed of $0.8c$?

Given Data:

$$t_0 = 20 \text{ min}$$

$$v = 0.8c$$

To find:

$$t = ?$$

Solution:

$$\begin{aligned} t &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{20}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = \frac{20}{\sqrt{\frac{c^2(1 - (0.8)^2)}{c^2}}} \\ &= \frac{20}{\sqrt{1 - 0.64}} = \frac{20}{\sqrt{0.36}} = \frac{20}{0.6} = 33.3 \text{ min} \\ \mathbf{t} &= \mathbf{33.3 \text{ min}} \end{aligned}$$

11.3 A spaceship is measured 100 m long while it is at rest with respect to an observer, if this spaceship now flies by the observer with a speed of $0.99c$, what length will the observer find for the spaceship?

Given Data:

$$\ell_0 = 100 \text{ m}$$

$$v = 0.99c$$

To find:

$$\ell = ?$$

Solution:

$$\ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2}} = 100 \times \sqrt{1 - \frac{(0.99c)^2}{c^2}} = 100 \times \sqrt{\frac{c^2(1 - (0.99)^2)}{c^2}}$$

$$\ell = 100 \times \sqrt{1 - (0.99)^2} = 100 \times \sqrt{1 - 0.9801}$$

$$\ell = 100 \times \sqrt{0.0199} \approx 100 \times 0.141 = 14.1 \text{ m}$$

$$\ell = \mathbf{14.1 \text{ m}}$$

11.4 The rest mass of an electron is $9.11 \times 10^{-31} \text{ kg}$. Calculate the corresponding rest mass energy.

Given Data:

$$m_0 = 9.11 \times 10^{-31} \text{ kg}$$

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

To find:

$$E_0 = ?$$

Solution:

$$E_0 = m_0 c^2$$

$$E_0 = 9.11 \times 10^{-31} \times (3 \times 10^8)^2 = 9.11 \times 10^{-31} \times 9 \times 10^{16} = \mathbf{8.2 \times 10^{-14} \text{ J}}$$

$$\text{or } E_0 = \frac{8.2 \times 10^{-14}}{1.6 \times 10^{-13}} \approx 0.51 \text{ MeV}$$

$$E_0 \approx \mathbf{0.51 \text{ MeV}}$$

11.5 An electron is accelerated to a speed $v = 0.85c$. Calculate its total energy and kinetic energy in electron volt.

Given Data

$$v = 0.85c$$

$$E_0 = m_0 c^2 = 0.511 \text{ MeV}$$

To find:

$$E = ?$$

$$\text{K.E.} = ?$$

Solution:

$$\text{Total Energy } E = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{0.511}{\sqrt{1 - \frac{(0.85c)^2}{c^2}}} = \frac{0.511}{\sqrt{\frac{c^2(1 - (0.85)^2)}{c^2}}} = \frac{0.511}{\sqrt{1 - 0.7225}} = \frac{0.511}{\sqrt{0.2775}}$$

$$E = \mathbf{0.97 \text{ MeV}}$$

Now, Kinetic Energy in MeV

$$\text{K.E.} = E - m_0 c^2 = 0.970 - 0.511 = 0.459 \text{ MeV}$$

$$\text{K.E.} = E - m_0 c^2 = 0.970 - 0.511 = 0.459$$

$$\text{K.E.} = \mathbf{0.459 \text{ MeV}}$$

11.6 At what speed, would the mass of a proton in a particle accelerator be tripled?

Given Data:

$$m = 3m_0$$

To find:

$$v = ?$$

Solution:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3m_o = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = 1/3 = \sqrt{1-\frac{v^2}{c^2}}$$

$$(1/3)^2 = 1 - v^2/c^2 \Rightarrow v^2/c^2 = 1 - 1/9 = 8/9$$

$$v = c \times \sqrt{\frac{8}{9}} \approx 0.943c$$

$$v = 0.943 c$$

11.7 The period of pendulum is measured to be 3 s in an inertial frame of reference. What will be the period measured by an observer in a spaceship with a constant speed of 0.95c with respect to the pendulum?

Given Data:

$$t_o = 3 \text{ s}$$

$$v = 0.95c$$

To find: $t=?$

Solution:

As,

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} = \frac{3}{\sqrt{1-\frac{c^2(1-(0.95)^2)}{c^2}}} = \frac{3}{\sqrt{1-0.9025}} = \frac{3}{\sqrt{0.0975}}$$

$$t = 3 / 0.3122 = 9.6 \text{ s}$$

$$t = 9.6 \text{ s}$$

11.8 Hypothetically, if a ball of mass 0.5 kg is projected with a velocity of 0.9c, what will be its mass in flight?

Given Data: $m_o = 0.5 \text{ kg}$

$$v = 0.9c$$

$$c = 3 \times 10^8 \text{ m/s}$$

To find: $m = ?$

Solution:

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0.5}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} = \frac{0.5}{\sqrt{1-\frac{c^2(1-(0.9)^2)}{c^2}}} = \frac{0.5}{\sqrt{1-0.81}} = \frac{0.5}{\sqrt{0.19}} = 1.146 \text{ kg}$$

$$m \approx 1.15 \text{ kg}$$

Chapter 12

Nuclear and Particle Physics

Exercise

MCQs

- 12.1** Which one of the following is the fundamental particle?
(a) Proton (b) Neutron (c) Electron (d) Meson
- 12.2** The first discovered anti-particle is:
(a) anti-proton (b) anti-neutrino (c) anti-photon (d) anti-electron
- 12.3** Which one of the following pair of particles creates annihilation?
(a) proton–proton (b) proton–neutron (c) neutron–photon (d) electron–positron
- 12.4** The strong nuclear force between the two particles is mediated by:
(a) gluons (b) photon (c) mesons (d) gravitons
- 12.5** Which one of the following forces interacts between two particles through photons?
(a) Strong nuclear force (b) Weak force
(c) Electromagnetic force (d) Gravitational force
- 12.6** When a neutron changes into a proton, then we will observe:
(a) β^- -decay (b) β^+ -decay (c) γ -decay (d) α -decay
- 12.7** Baryon is formed by combination of:
(a) 2 quarks (b) 3 quarks (c) 4 quarks (d) A quark and an anti-quark
- 12.8** Which one of the following forces has negligible effect between the elementary particles?
(a) Strong nuclear force (b) Weak force
(c) Gravitational force (d) Electromagnetic force
- 12.9** Which particles are produced by strong interaction?
(a) Graviton (b) Leptons (c) Hadrons (d) Mesons
- 12.10** A strong nuclear force exists between the nucleons of:
(a) p–p (b) n–n (c) p–n (d) all of these
- 12.11** Which one of the following radiation/particles has the highest ionization power?
(a) α (b) β^+ (c) β^- (d) γ
- 12.12** Which one of the following radiation/particles has the highest penetrating power?
(a) α (b) β^+ (c) β^- (d) γ
- 12.13** A change occurs in atomic number of a nucleus but its mass number remains the same by decay of:
(a) α (b) β (c) γ (d) α and γ
- 12.14** In a nucleus, a neutron changes into a proton, the atomic number changes by one, the mass number will:
(a) decrease (b) increase (c) remain the same (d) none of these
- 12.15** The electroweak theory was introduced by:
(a) Dirac (b) Einstein (c) Anderson (d) Dr. Abdul Salam
- 12.16** The asymmetry of matter and anti-matter is due to imbalance number of:
(a) hadron (b) lepton (c) baryon (d) photons
- 12.17** Which one of the following particle is responsible for the mass of the fundamental particle?
(a) Quarks (b) Anti-quark (c) Lepton (d) Higgs boson
- 12.18** A proton is composed of up and down quarks, the order of quarks is:
(a) udd (b) udu (c) uud (d) dud
- 12.19** The number of quarks that composed of a neutron is:
(a) 2 (b) 3 (c) 4 (d) 5

Answer Key

Q.No.	Option	Q.No.	Option	Q.No.	Option	Q.No.	Option	Q.No.	Option
12.1	C	12.5	C	12.9	C	12.13	B	12.17	D
12.2	D	12.6	A	12.10	D	12.14	C	12.18	C
12.3	D	12.7	B	12.11	A	12.15	D	12.19	B
12.4	A	12.8	C	12.12	D	12.16	C		

Short Answer Questions

12.1 What do different isotopes of a given element have in common? How are they different?

Ans Isotopes have the same number of protons but different number of neutrons. Also, they have the same chemical properties but different physical properties.

Example. $^{12}_6\text{C}$, $^{13}_6\text{C}$ and $^{14}_6\text{C}$ are three isotopes of carbon.

12.2 Identify the elements that has 87 nucleons and 50 neutrons.

$$A = 87$$

$$N = 50$$

$$Z = A - N = 87 - 50$$

$$Z = 37$$

An element with atomic number 37 is Rubidium ($^{87}_{37}\text{Rb}$).

12.3 What are similarities and differences between the strong nuclear force and the electromagnetic force?

Similarities:

- Both are fundamental forces of nature.
- Both are mediated by exchange particles (gluons and photons).

Differences:

Strong Nuclear Force	Electromagnetic Force
Short-range force	Long-range force
Acts only inside the nucleus and is attractive	Responsible for atomic structure and chemical bonding
It is attractive	It is attractive or repulsive

12.4 Fill in the missing particle or nucleus:

(a) $^{45}_{20}\text{Ca} \rightarrow ? + e^- + \bar{\nu}$

(b) $^{58}_{29}\text{Cu} \rightarrow ? + \gamma$

Ans (a) $^{45}_{20}\text{Ca} \rightarrow ? + e^- + \bar{\nu}$

$^{45}_{20}\text{Ca} \rightarrow ^{45}_{21}\text{Sc} + e^- + \bar{\nu}$,

So missing particle is scandium $^{45}_{21}\text{Sc}$

(b) $^{58}_{29}\text{Cu} \rightarrow ? + \gamma$

$^{58}_{29}\text{Cu}^* \rightarrow ^{58}_{29}\text{Cu} + \gamma$

So the missing particle is copper ($^{58}_{29}\text{Cu}$)

12.5 Why neutrino must be released in the positron emission.

Ans As neutrino carries energy and momentum during positron emission. Neutrino are released to conserve energy, momentum, and spin during the decay process.

12.6 Distinguish between fermions and bosons.

Fermions: These are the particles which follow the Pauli exclusion principle.

They have half integer spin (like electrons, protons, neutrons).

Bosons: These are the particles which do not follow the Pauli exclusion principle.

These particles have zero and whole number spin (like photons, gluons).

12.7 How does strong force hold the nucleus?

Ans The strong force binds protons and neutrons together by overcoming the repulsive electromagnetic force between protons. It acts only for very short distances inside the nucleus.

12.8 Can there be pair production for photons having energy 20 keV? Explain briefly.

Ans No, pair production cannot take place with 20 keV energy.

It needs at least 1.02 MeV energy, 20 keV is too low to create an electron-positron pair.

12.9 What is the difference between beta particle and electron?

Ans A beta particle is a high-energy electron emitted from the nucleus during beta decay.

While an electron is a fundamental particle orbiting the nucleus and having less energy than beta particle.

12.10 How do a proton and a neutron convert to each other?

Ans

- Neutron $\rightarrow$ Proton + Electron + Antineutrino (Beta-minus decay)
- Proton $\rightarrow$ Neutron + Positron + Neutrino (Beta-plus decay)

12.11 Why does beta-decay have a continuous energy spectrum and alpha-decay have a discrete energy spectrum?

Ans In beta-decay, energy is shared between beta particles and anti-neutrino in varying proportions. Due to this, it has continuous spectrum.

In alpha-decay, every alpha particle is emitted has the same sharply defined kinetic energy. When the number of alpha particles is plotted against kinetic energy there are distinct spikes that appear on the graph. It demonstrates that alpha particles have discrete energies.

12.12 Differentiate between hadron and leptons with examples.

Ans

- **Hadrons** experience strong nuclear force (e.g., proton, neutron).
- **Leptons** do not experience strong nuclear force (e.g., electron, neutrino).

12.13 Why electron-positron pair cannot decay into a single photon?

Ans A single photon cannot conserve energy and momentum together in this decay. At least two photons are needed. Creation of single photon from electron positron pair is against the law of conservation of energy and momentum.

12.14 State the role of Higgs Boson in the generation of mass in modern physics theories.

Ans Higgs Boson is responsible for giving mass to particles through the Higgs field, according to the Standard Model.

12.15 What are mesons? Give example.

Ans Mesons are sub-atomic particles made of a quark and an antiquark.

Example: Pion (π^+ , π^0 , π^-).

Constructed Response Questions

12.1 Is meson, a boson or fermion? Give reason.

Ans Meson is a boson.

Since mesons are composed of one quark and one anti-quark with spin values (0, 1, 2, ...). This is also characteristic of bosons.

12.2 Why does an alpha emitter emit alpha particles instead of four separate nucleons?

Ans Alpha particles have greater nuclear stability as compared to individual nucleons, i.e., alpha particle nucleons are strongly held together that makes the nucleus more stable than individual nucleons. Therefore, alpha emitter emits alpha particles instead of four separate nucleons.

12.3 Which is more energetic alpha decay or beta decay? Justify your answer.

Ans Alpha decay is more energetic than beta decay.

As, mass-energy relation is:

$$E = \Delta mc^2$$

$$E \propto \Delta m$$

Alpha particle is more massive than beta particle. Mass difference between reactants and products is much greater during beta decay.

12.4 A nucleus undergoes gamma decay, emitting gamma ray photon with energy 1.5 MeV. Calculate.

(i) frequency of gamma ray

(ii) Wavelength of gamma ray

(iii) momentum of gamma ray

Ans (i)

$$E = 1.5 \text{ MeV} = 1.5 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$E = 2.4 \times 10^{-13} \text{ J}$$

$$f = ?$$

$$E = hf$$

$$f = \frac{E}{h} = \frac{2.4 \times 10^{-13}}{6.63 \times 10^{-34}} = 3.63 \times 10^{20} \text{ Hz}$$

(ii)

$$E = 1.5 \text{ MeV} = 1.5 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$E = 2.4 \times 10^{-13} \text{ J}$$

$$\lambda = ?$$

$$E = hf = \frac{hc}{\lambda} \quad \because c = f\lambda$$

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.4 \times 10^{-13}} = 8.27 \times 10^{-13} \text{ m}$$

(iii)

$$E = 1.5 \text{ MeV} = 1.5 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$E = 2.4 \times 10^{-13} \text{ J}$$

$$p = ?$$

$$E = cp$$

$$p = \frac{E}{c} = \frac{2.4 \times 10^{-13}}{3 \times 10^8} = 8.01 \times 10^{-22} \text{ kg m s}^{-1}$$

12.5 Why does the α -particles not make physical contact with the nucleus when headed directly towards it?

Ans As both alpha particles and the nucleus are positively charged. Therefore, alpha particles do not make physical contact with the nucleus when headed directly toward it due to electrostatic force of repulsion.

Comprehensive Questions

12.1 What is meant by radioactivity? Compare the properties and behavior of three types of radiations.

Ans Radioactivity

It has been observed that the nuclei whose atomic numbers are greater than 82 are found naturally unstable, these nuclei spontaneously emit radiations. Such nuclei are called radioactive and the emission of radiation is known Radioactivity.

Artificial Radioactivity:

When a stable element is bombarded with high energy particles such as neutrons, protons, or alpha particles, it becomes unstable and emits radiations. This is called artificial radioactivity and the radioactive isotopes are called radioisotopes or radionuclides.

Nuclear Radiations

Nuclear radiations are of three types α , β and γ -radiations.

Properties of α -particles, β -particles and γ -radiations

1. **Alpha (α) Particles**

- Alpha particle is a helium nucleus (${}^4_2\text{He}$) consisting of 2 protons and 2 neutrons.
- They carry positive (+2) charge on it.
- They have large mass.
- They are deflected towards the negative terminal of the electric field, which shows they have a positive charge.
- They travel only several centimeters in air, due to their large mass.
- They have the highest ionization power and lowest penetration power among the three radiations.
- In the alpha decay, atomic number of parent nuclei decreases by 2 and mass number decreases by 4.

2. **Beta (β) Particles**

- β particles are fast-moving electrons; created at the moment of emission.
- They carry negative (-1) charge, equal to that of electron.
- They have very small mass (much lighter than alpha particles).
- They have very high speed, up to 0.9995 of the speed of light.
- They deflect towards the positive terminal of the electric field, which shows they have negative charge.

- They show greater deflection than alpha particles, due to lower mass.
- They have moderate penetration power and ionizing power.
- During β^- decay, the atomic number of parent nuclei increases by 1, mass number remains the same.
- During β^+ decay, atomic number decreases by 1, mass number remains the same.

3. Gamma (γ) Radiations

- Gamma rays are electromagnetic radiations consisting of high-energy photons.
- They have no charge (neutral) and rest mass.
- They travel at the speed of light.
- They pass straight through the electric field, showing no deflection.
- They have highest penetrating power and lowest ionization power.
- During gamma decay the parent nuclei shows no change in atomic number (Z) and mass number (A).

12.2 Elaborate the phenomenon of beta-positive decay and beta-negative decay with examples.

Ans Beta Decay

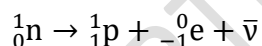
There are two types of β -decay: β^- -decay and β^+ decay.

(i) β^- -Decay

Some nuclides have neutron-to-proton ratio (N/P) too large and are a source of β^- -decay.

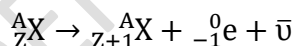
β particles are not orbital electrons; they are created within nucleus at the time of emission.

In this process, a neutron in the nucleus decays into a proton an electron, and an antineutrino according to following relation:



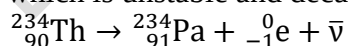
One of the neutron changes into a proton and in order to conserve charge it emits an electron. These emitted electrons are called beta particles. They are identical to orbital electrons. No nucleons are lost during β^- -Decay; the mass number remains same, but the atomic number increases by 1.

General equation:



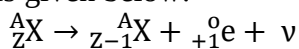
Example:

An example is the isotope of thorium, which is unstable and decays into protactinium by beta emission.



(ii) Beta-Positive Decay:

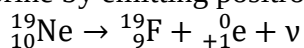
Some nuclides have neutron-to-proton ratio (N/P) too low for stability. They decay by emitting a positron and a neutrino. Positron (e^+) has the same mass as the electron, but it has a positive charge. It is antiparticle of electron. In this process, a proton in the nucleus decays into a neutron and a positron plus a neutrino. The generalized decay is given below:



The atomic number decreases by 1, while the mass number stays the same.

Example:

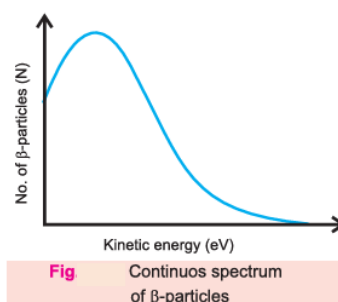
An example of decay of Neon into Fluorine by emitting positron and neutrino is:



Energy of Beta Particles in Radioactive Decay

In beta decay, energy is shared between β -particles and anti-neutrino in varying proportions. The sum of electron (or positron) energy and anti-neutrino's (or neutrino's) energy, remains the same. Thus, in β decay, the energy of an electron or a positron may range from zero to maximum value.

When the number of β particles is plotted against kinetic energy, the graph demonstrates that beta particles have continuous range of energies. The principle of conservation of momentum and energy applies in beta emission.



12.3 What is the difference between matter and anti-matter? Discuss reasons why our universe is almost entirely composed of matter.

Ans Matter and Antimatter

Matter is made up of elementary particles such as electrons, protons, and neutrons that form everything we see around us. Anti-matter consists of anti-particles that are exact opposites of these elementary particles.

It was predicted by Paul Dirac in 1928 that the fundamental particles have their anti-particles. The difference is that the rest masses of the anti-particles are the same but with opposite charges and magnetic moments.

For example:

Positron is the anti-particle of electron. It is represented by e^+ . Its rest mass is same as electron but it carries positive charge with same magnitude as an electron.

Positron was the first discovered anti-particle by Anderson in 1932 in a cloud chamber experiment. This was the first experimental discovery of an anti-particle. After that a lot of anti-particles were discovered. The proton has an anti-particle called the antiproton, which has the same mass but a negative charge. Quarks and leptons, which have been recognized as the fundamental (elementary) particles, also have their anti-particles.

Usually, the anti-particles are represented by a letter with a bar over it, e.g., anti-proton is represented by $\bar{p}$ and anti-neutrino by $\bar{\nu}$.

Universe is Almost Entirely composed of Matter

Our universe is very vast; we still know about its 5% part but 95% part is still unknown. For example, our universe consists of about 27% of unknown matter called dark matter and about 68% a mysterious antigravity material known as dark energy. By adding 27% and 68% we have 95%. It means we know nothing about 95% of the universe.

On the other hand, the 5% of the universe that we know about it, is an ordinary matter, where hydrogen and helium are almost in plasma state. Therefore, these figures indicate that 95% of the 5% of the observable universe is the plasma state, while the remaining 5% is in the form of matter.

12.4 Explain the phenomenon of pair annihilation with an example. Explain the utility of its principle in the medical field.

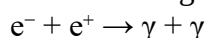
Ans Pair Annihilation (Annihilation of Matter)

Pair annihilation is a process in which a particle and its corresponding anti-particle collide and completely destroy each other, converting their entire combined rest mass into energy. The energy is usually released in the form of gamma-ray photons.

It is the reverse process of pair production.

The reaction conserves both energy and momentum.

Example: A common example is the interaction of an electron (e^-) and a positron (e^+). They annihilate into two gamma ray photons as shown in figure. The reaction can be written as:



Each gamma-ray photon carries 0.51 MeV of energy, which is equal to the rest mass energy of an electron or a positron i.e., $E = m_0c^2$. In an annihilation reaction, energy and momentum are conserved.

Besides the electron and positron annihilation, the annihilation reactions of other particles and their anti-particles can also be carried out e.g., proton and antiproton, quark and antiquark, lepton and anti-lepton.

Utility of pair annihilation in the medical field

Pair annihilation is a critical practical tool in modern medicine.

1. Positron Emission Tomography (PET) Imaging: PET is the most significant clinical application of pair annihilation, which provides information at functional and molecular level.

It applies to

- (i) Cancer Detection and staging.
- (ii) Monitoring Therapy Response.
- (iii) Neurological Imaging.
- (iv) Cardiology.

2. Emerging Cancer Treatment (pair-production Radiotherapy): While typical X-ray therapies use lower energy photons, advanced linear accelerators can produce high-energy photons ($>1.022 \text{ MeV}$) that induce electron-positron pair production within tumors.

It applies to:

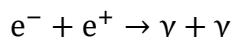
- (i) Enhanced Tumor Destruction.
- (ii) Pair Production Tomography.

12.5 Explain the law of conservation of energy and momentum in electron-positron pair annihilation.

Ans Law of Conservation of Energy in Electron-Positron Pair Annihilation:

The law of conservation of energy states that the total energy before and after any physical process must remain the same; energy can neither be created nor destroyed, only transformed from one form to another.

The electron-positron pair annihilation can be expressed as:



In this reaction, before annihilation, the total energy consists of the rest mass energy of the electron plus the rest mass energy of the positron, plus any kinetic energy they may have. The rest mass energy of an electron or positron is 0.51 MeV each.

Therefore, the total rest mass energy is $0.51 \text{ MeV} + 0.51 \text{ MeV} = 1.02 \text{ MeV}$.

After annihilation, the two particles disappear, and their entire mass is converted into energy carried away by two gamma-ray photons. Each gamma-ray photon has an energy of 0.51 MeV .

Thus, the total energy of the two photons is $0.51 \text{ MeV} + 0.51 \text{ MeV} = 1.02 \text{ MeV}$.

Hence,

The total energy before annihilation = the total energy after annihilation.

Law of Conservation of Momentum in Electron-Positron Pair Annihilation:

The law of conservation of momentum states that the total momentum of a closed system before and after any interaction must remain the same.

Before annihilation, the total momentum of the system is approximately zero.

After annihilation, two gamma-ray photons are produced. Photons have zero rest mass however they carry momentum. The two photons travel in exactly opposite directions i.e. their momenta are equal in magnitude but opposite in direction.

Therefore,

The total momentum before annihilation = total momentum after annihilation (i.e. ≈ 0).

This is why electron-positron annihilation produces two gamma rays i.e. to conserve both energy and momentum in the process.

12.6 Describe protons and neutrons in terms of their quark composition.

Ans Protons and Neutrons in Terms of Their Quark Composition

In 1964, M. Gell Mann and George Zweig proposed that all hadrons, including baryons: protons and neutrons, are made up of fundamental entities called quarks.

Each baryon consists of three quarks bound together by the strong force.

Proton

The proton has a quark composition: uud (two up quarks and one down quark)

So its charge quantum number is:

$$q(uud) = 2u + 1d$$

$$q(uud) = \frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1$$

Neutron

The neutron has a quark composition: udd (one up quark and two down quarks)

So its charge quantum number is:

$$q(udd) = 1u + 2d$$

$$q(udd) = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$



After the quark theory was proposed, it was suggested that quarks have another property called color or **color charge**. According to this theory, each quark can have one of three colors, red, blue or green.

Baryons (proton and neutron) are made up of three quarks, one of each color and are thus colourless or white. Each quark is assumed to carry a colour charge, and the strong force between quarks is referred to as the color force according to the quantum chromodynamics (QCD).



Fig. : Colourless baryons:
blue + red + green = white

Thus, protons and neutrons are not fundamental particles but composite structures made of up and down quarks in specific combinations, explaining their similar masses and their differing electric charges.

12.7 Describe four fundamental forces in nature.

Ans Four Fundamental Forces in Nature

All interactions in the universe are governed by four basic or fundamental forces of nature. These forces control how objects move, interact and behave at different scales from nucleons in the atom to massive galaxies. The four fundamental forces are:

1. Gravity
2. Electromagnetism
3. Weak Nuclear Force
4. Strong Nuclear Force

1. Gravitational Force

Gravitational force or gravity is the weakest of the four fundamental forces but it is long-range force. It is an attractive force that arises due to interaction between objects. The gravitational force is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Gravity becomes noticeable and significant when dealing with objects of very large mass, such as giant planets, stars, and galaxies. At atomic and subatomic scales, gravitational force is extremely weak and negligible.

2. Electromagnetic Force:

The electromagnetic force is responsible for all electric and magnetic interactions. It is much stronger than gravity (about 10^{36} times stronger) and also follows an inverse square law like gravitational force. This force is responsible for a wide range of phenomena, including:

- (i) Atomic structure: Holding electrons in orbit around the atomic nucleus
- (ii) Chemical bonding between atoms
- (iii) Electricity and magnetism, and
- (iv) the propagation of light and all electromagnetic waves

James Clerk Maxwell unified electricity and magnetism into a single electromagnetic force through his four fundamental equations in 1861. showing that electric and magnetic fields are two aspects of the same force.

3. Strong Nuclear Force:

The strong nuclear force is the strongest attractive force. It is responsible for the stability of the nucleus. Without the strong force, the positively charged protons would repel each other due to electromagnetic force, causing the nucleus to disintegrate. The strong force overcomes this repulsion and binds all nucleons (protons and neutrons) together.

Key features of strong force are:

- (i) Strong nuclear forces are responsible for holding the nucleus.
- (ii) They exist only inside the nucleus.
- (iii) It is a very short-range force, (range less than about 10^{-15} m).
- (iv) Beyond this short distance, the force drops off rapidly to zero.
- (v) It is as an attractive force between all nucleons.
- (vi) Protons attract by strong nuclear force and repel each other by electric force.

4. Weak Nuclear Force:

The weak nuclear force is responsible for processes that involve a change in the identity of particles, especially in radioactive decay and neutrino interactions.

It plays a key role in:

- (i) Nuclear fusion reactions in stars
- (ii) Decay of unstable atomic nuclei
- (iii) Beta decay

Unlike the other forces, the weak force can transform one type of particle into another (e.g., changing a down quark into an up quark). It is much weaker than the strong and electromagnetic forces but stronger than gravity, and it operates over an extremely short range ($< 10^{-18}$ m).

Summary of the Four Fundamental Forces

Force	Approximate Relative Strength (compared to strong force)	Range	Key Characteristics and Role
Strong Nuclear Force	1 (strongest)	$< 10^{-15}$ m	Binds protons and neutrons in the nucleus; short-range; attractive between all nucleons
Electromagnetic Force	10^{-2}	∞	Governs electric/magnetic interactions, atomic structure, chemistry, light; long-range
Weak Nuclear Force	10^{-13}	$< 10^{-18}$ m	Responsible for beta decay, neutrino interactions, particle identity changes; very short-range
Gravitational Force	10^{-38} (weakest)	∞	Attractive force between masses; significant only for large masses (planets, stars); long-range

12.8 Describe the classification of elementary particles.

Ans Elementary Particles:

Elementary (fundamental) particles are those that have no internal structure, meaning they are indivisible. The currently accepted fundamental constituents of matter are quarks and leptons. They are considered as the basic building blocks of matter.

Quarks: Protons, neutrons, and mesons are made up of quarks.

Leptons: Electrons, positrons and neutrinos are included in leptons.

Classification of Elementary Particles:

Particles are classified based on their:

1. spin behavior,
2. nucleon interaction
3. strong nuclear force interaction

1. Spin behavior

All particles spin on their axes and the spin of charged particles makes them tiny magnets.

The characteristic spin of electrons, protons and neutrons is $\frac{1}{2}$; spin of photons is 1 and of pions is 0.

(i) Fermions:

The particles with half-integer spin ($\frac{1}{2}$, $\frac{3}{2}$, etc.) are called fermions. They obey **Pauli's exclusion principle** (i.e. only one particle of a kind can occupy a given quantum state). They follow Fermi-Dirac statistics. All fundamental fermions (quarks and leptons) have spin $\frac{1}{2}$.

Examples: electrons, quarks, protons, neutrons, neutrinos.

(ii) Bosons:

The particles with integer spin (0, 1, 2, etc.) or zero spin are called bosons. They do not obey Pauli's exclusion principle. They follow Bose-Einstein statistics.

Examples: photons (spin 1), pions (spin 0).

2. Nucleon Interaction

(i) Baryons:

The nucleons, and the heavier particles which decay to nucleons, are called baryons (heavy).

Examples: protons, neutrons, and heavier particles (such as Λ^0 and K^0) that decay into nucleons.

(ii) Leptons:

The particles that do not interact strongly with nucleons are called leptons (small).

Examples: electrons, tau, neutrino

3. Interaction with the Strong Nuclear Force

Particles can be broadly classified based on whether they interact via the strong force or not.

(i) Hadrons:

The particles that interact via the strong nuclear force are known as hadrons.

Examples: protons, neutrons, and pions.

Hadrons are composite subatomic particles that can be subdivided into two broad categories:

(a) Baryons: They are made of an odd number of quarks (usually three quarks). They are fermions (half-integer spin). heavier particles (such as Λ and Σ) that decay into nucleons.

Examples: protons, neutrons

(b) Mesons: They are made up of an even number of quarks (usually two quarks: one quark and one antiquark). They are bosons (integer spin).

Examples: pions (π mesons), kaons (K mesons).

(ii) Leptons:

The particles that do not interact via the strong nuclear force are known as leptons.

Examples: electrons, and neutrinos

They interact only through the weak force or electromagnetic force.

Leptons appear to be truly fundamental particles that cannot be split into smaller particles. All leptons have spin $\frac{1}{2}$ so they are fermions. The six known leptons are grouped into three pair of particles, each containing:

(i) one charged particle (electron (e^-), muon (μ^-), tau (τ^-))

(ii) one associated neutrino (electron neutrino, muon neutrino, tau neutrino)

(iii) and their antiparticles.

This classification shows that only quarks and leptons are truly elementary, while hadrons are composite structures built from quarks. The distinction between hadrons and leptons is primarily based on whether they participate in the strong nuclear force.

Numerical Problems

12.1 Uranium-238 is an alpha emitter. In the process, it is transmuted into a daughter nucleus. What is the mass number A and charge number Z of the daughter nucleus? What is its chemical symbol?

Given Data:

Parent nucleus = ${}_{92}^{238}\text{U}$

To find

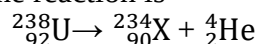
Daughter nucleus = Y = ?

Chemical Symbol of Daughter nucleus = ?

Solution:

During alpha emission, A decreases by 4 and Z decreases by 2,

Thus the reaction is



As, for Thorium, A = 234 and Z = 90

So, ${}_{92}^{238}\text{U} \rightarrow {}_{90}^{234}\text{Th} + {}_2^4\text{He}$

Therefore, the daughter nucleus has

Mass number A = 234

Charge number Z = 90

Chemical Symbol: Thorium (Th)

12.2 Polonium $^{218}_{84}\text{Po}$ is a beta-minus emitter. What will be the mass number A and charge number Z of the daughter nucleus?

Given Data:

$$\text{Parent nucleus} = {}^{218}_{84}\text{Po}$$

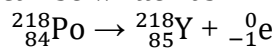
To find

$$\text{Daughter nucleus} = Y = ?$$

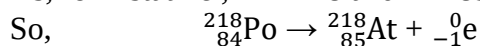
Solution:

Beta-minus decay emits ${}^0_{-1}\text{e}$, therefore, Z is increased by 1 and A remains same.

Thus reaction can be written as



As, for Astatine, A = 218 and Z = 85



Therefore, the daughter nucleus has

Mass number A = 218

Charge number Z = 85

12.3 Nitrogen $^{14}_7\text{N}$ bombarded by alpha particle results into $^{17}_8\text{O}$. What is the product particle in this nuclear reaction?

Given Data:

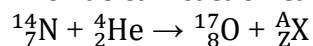
$^{14}_7\text{N}$ is bombarded by alpha particle and resultant nucleus = $^{17}_8\text{O}$

To find:

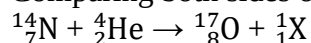
$$\text{Product particle} = X = ?$$

Solution:

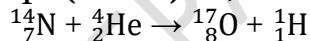
The nuclear reaction can be expressed as



Comparing both sides of the reaction,



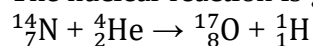
Hence, the missing particle is: ${}^1_1\text{H}$ (Proton) i.e.,



12.4 Show that nucleon number N and charge number Z are conserved in numerical question 12.3.

Solution:

The nuclear reaction is given by



Comparing both sides of the reaction,

$$\text{Nucleon number A: } 14 + 4 = 18 \rightarrow 17 + 1 = 18$$

$$\text{Charge number Z: } 7 + 2 = 9 \rightarrow 8 + 1 = 9$$

Hence, nucleon number N and charge number Z are **conserved** in the nuclear reaction.

12.5 Determine the rest mass energy of an electron in eV. Its rest mass is 0.000555 u.

Given Data:

$$\text{Rest mass of electron} = 0.000555 \text{ u}$$

To find

$$\text{Rest mass energy of electron} = ?$$

Solution:

$$\text{As, } E = mc^2$$

$$E = (0.000555 \text{ u}) (3 \times 10^8)^2$$

$$= (0.000555) (1.6606 \times 10^{-27} \text{ kg}) (9 \times 10^{16}) \quad (\because 1 \text{ u} = 1.6606 \times 10^{-27} \text{ kg})$$

$$= 82.94 \times 10^{-15} \text{ J}$$

$$= \frac{82.94 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 51.7 \times 10^4 \text{ eV}$$

$$(\because 1 \text{ J} = \frac{1}{1.6 \times 10^{-19}} \text{ eV})$$

$$E = 0.517 \text{ MeV}$$

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12.6 Calculate the Q-value for the reaction taking place in Rutherford's experiment on artificial disintegration of nitrogen by bombardment with alpha particles. Relative masses are:

$^{14}\text{N}_7 = 14.007515 \text{ u}$, $^4\text{He}_2 = 4.003837 \text{ u}$, $^{17}\text{O}_8 = 17.004533 \text{ u}$, and $^1\text{H}_1 = 1.008142 \text{ u}$.

Given data:

(Relative mass of $^{14}\text{N}_7$) = 14.007515 u

(Relative mass of $^4\text{He}_2$) = 4.003837 u

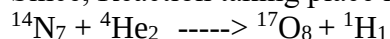
(Relative mass of $^{17}\text{O}_8$) = 17.004533 u

(Relative mass of $^1\text{H}_1$) = 1.008142 u

To find:

Q-value = ?

Solution: Since, Reaction taking place in Rutherford's experiment is



As, Q-value = $(\Delta m)c^2$ (i)

Where, Δm = (Sum of masses of reactant) – (Sum of masses of products)

$$= (14.007515\text{u} + 4.003837\text{u}) - (17.004533\text{u} + 1.008142\text{u})$$

$$= 18.011352\text{u} - 18.012675\text{u}$$

$$= -0.001323\text{u}$$

$$= -0.001323 \times 1.6606 \times 10^{-27} \text{ Kg}$$

$$1\text{u} = 1.6606 \times 10^{-27} \text{ Kg}$$

$$= -1.323 \times 10^{-3} \times 1.6606 \times 10^{-27} \text{ Kg}$$

$$= -2.197 \times 10^{-30}$$

Therefore, equation (i) becomes

$$\text{Q-value} = (-2.197 \times 10^{-30})(3 \times 10^8)^2$$

$$= (-2.197 \times 10^{-30})(9 \times 10^{16})$$

$$= -19.773 \times 10^{-15} \text{ J}$$

$$= (-19.773 \times 10^{-14}) / (1.6 \times 10^{-19}) \text{ eV}$$

$$= -12.3 \times 10^5 \text{ eV}$$

$$= -1.23 \times 10^6 \text{ eV}$$

$$= -1.23 \text{ MeV}$$